Descriptive Profiles for Sets of Alternatives in Multiple Criteria Decision Aid

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Abstract. In the context of Multiple Criteria Decision Aid, a decisionmaker may be faced at any time with the task of analysing one or several sets of alternatives, irrespective of the decision he is about to make. As in this case the alternatives may express contrasting gains and losses on the criteria on which they are evaluated, and while the sets that are presented to the decision-maker may potentially be large, the task of analysing them becomes a difficult one. Therefore the need to reduce these sets to a more concise representation is very important.

Classically, profiles that describe sets of alternatives may be found in the context of the sorting problem, however they are either given beforehand by the decision-maker or determined from a set of assignment examples. We would therefore like to extend such profiles, as well as propose new ones, in order to characterise any set of alternatives. For each of them, we present several approaches for extracting them, which we then compare with respect to their performance.

Keywords: multiple criteria decision aid, descriptive profiles, central profiles, bounding profiles, separating profiles, outranking relations, meta-heuristics.

1 Introduction

The field of Multiple Criteria Decision Aid (MCDA) focuses on modelling the preferences of decision-makers and aims at helping them in reaching certain decisions. This is not an easy task, as the entities that make the object of these decisions are defined on multiple dimensions and in many cases express contrasting gains and losses on them. Many different models have been proposed in order to reflect the subjective perspective of the decision-maker (DM) over these entities, or alternatives. Hence we are able to distinguish at least three types of relations between them [7]: indifference, strict preference and incomparability.

Classically, three well known types of decision problems have been defined in MCDA [9]: choice, ranking and sorting. The first consists in finding the best alternative, or the set containing the best ones, the second looks to build an order, partial or weak, over the set of decision alternatives, while the last problem outputs an assignment of the alternatives to a predefined set of classes, which may

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be ordered or not. Furthermore, another type of problem, that of clustering, has begun to receive increasing interest recently, having been specifically redefined in the context of MCDA [5].

In all of the mentioned MCDA problem types we may be faced at some point with one or several sets of alternatives, either as a final recommendation or during the decision aiding process. Due to the multidimensional nature of these alternatives and the possibility of having large sets, being able to describe them using concise information becomes very important. In the case of the sorting problem, the profiles that are used to describe the classes already serve this purpose, however this is not the case for the other types of problems. We mention the central profiles [6,1] that are used for sorting into nominal classes, as well as the delimiting profiles [11,9] for sorting into ordered classes.

In this paper we extend the profiles that have been used in conjunction with the problem of sorting, as well as we propose new ones, in order to be able to reduce the information given by one or several sets of alternatives to a more concise representation. These profiles may then be used in order for the DM to better understand the sets of alternatives that he is confronted with, while considering his preferences over them.

We begin by first defining the proposed profiles in Section 2. We consider only the case where a preference model is based on an outranking relation [8]. In Section 3 we first present several exact approaches for extracting these profiles. In order to deal with complexity issues that would be faced when the sets of alternatives are of large cardinality, we especially focus on the use of meta-heuristic approaches for constructing these descriptive profiles. The proposed approaches are validated and compared in Section 4 over a large set of benchmarks that contain increasingly contrasting alternatives. Finally, we conclude with a series of remarks and perspectives for the presented work.

2 Defining the Profiles

Before defining the profiles we first present the working context. Let X be the set of all decision alternatives that can be constructed on a set of criteria $F = \{1, ..., p\}$. We denote with A and B two subsets of X and their cardinalities with n and m. The evaluation of any alternative $x \in X$ on any criterion $i \in F$ is denoted by x_i .

We consider that the DM's preferences are modelled using an outranking relation, which we denote with S [8]. For the purpose of formally defining the profiles, any outranking relation may be considered. While an outranking relation is used to reflect whether the DM considers an alternative to be at least good as another, we may additionally use it to express judgements with respect to the notions of indifference, strict preference or even incomparability [7]. Two alternatives x and y are thus indifferent if simultaneously x outranks y and youtranks x. An alternative x is said to be strictly preferred to another alternative y when x outranks y and y does not outrank x. These alternatives are incomparable when neither of them outranks the other. We will consider three profiles for characterising one or several sets of alternatives. These are the central, bounding and separating profiles.

We define a **central profile** for a set A as an alternative, real or fictive, which is indifferent to as many of the alternatives in A as possible. Based on this definition, a central profile may be used to substitute all the alternatives in A, due to that fact that the DM considers it indifferent to them, therefore he cannot distinguish between them.

We model the fitness of a central profile c_A with respect to the set A through:

$$f^{c}(c_{A},A) = \frac{1}{n} \cdot \left| \left\{ x \in A \colon x \operatorname{I} c_{A} \right\} \right|$$

$$\tag{1}$$

The function above is straightforward, giving the proportion of alternatives in A that are indifferent to the central profile, inside a [0,1] value range.

When the alternatives in A are mostly indifferent to each other, a central profile may be able to represent them with a high degree of confidence. However, if we consider that the alternatives in A have rather contrasting evaluations, a central profile may not be able to represent them faithfully. For this purpose we will consider the second type of descriptive profile, the bounding profiles.

The bounding profiles may be seen as the best and worst alternatives in A, bounding all the rest between them with respect to the preferences of the DM.

We define the **upper bounding profile** of a set A as an alternative which is either strictly preferred or indifferent to any alternative in A, but not strictly preferred by them. In this way we may state that no alternative in A is better than the upper bounding profile. Similarly, the **lower bounding profile** of A is either strictly preferred by any alternative in A, or indifferent to them, therefore no alternative in A may be said to be worse than the lower bounding profile. We denote these profiles with b_A^+ and b_A^- respectively. When the upper or lower bounding profiles cannot be selected from A we may

When the upper or lower bounding profiles cannot be selected from A we may proceed to construct them using the following functions to model their quality:

$$f^{b+}(b_A^+, A) = \frac{1}{n \cdot (n+1)} \cdot \left(n \cdot \left| \{ x \in A \colon b_A^+ \operatorname{S} x \} \right| + \left| \{ x \in A \colon x \operatorname{S} b_A^+ \} \right| \right)$$
(2)

$$f^{b-}(b_A^-, A) = \frac{1}{n \cdot (n+1)} \cdot \left(\left| \{ x \in A \colon b_A^- \operatorname{S} x \} \right| + n \cdot \left| \{ x \in A \colon x \operatorname{S} b_A^- \} \right| \right)$$
(3)

Each of the two fitness measures counts the number of alternatives from A that the considered bounding profile outranks in the first term of the sum, but also the number of alternatives in A that are outranked by it in the second term.

Since an upper bounding profile mainly has to outrank all the alternatives in A (hence it will be either strictly preferred or indifferent to them), the first term has been weighted so that it dominates the second. We would also like to have an upper bounding profile that is indifferent to as many alternatives in Aas possible, therefore the second term is also necessary.

Similarly, the lower bounding profile reverses the importance of the two terms, as it mainly needs to be outranked by the alternatives in A (hence it will be either strictly preferred by them or indifferent). Nevertheless, if this first condition is met, then the lower bounding profile should also outrank the alternatives in A in order to be indifferent to them.

Finally, we consider two sets of alternatives, A and B, and a relation of strict preference of the first over the second. In such a case we may consider defining a profile that separates the alternatives between the two sets as well as possible.

We define a **separating profile** between a set A that is strictly preferred to a set B, as an alternative, real or fictive, which is strictly preferred by the alternatives in A, or at least indifferent to them, while in turn it is strictly preferred to the alternatives in B, or at least indifferent to them.

The fitness measure for such a profile is:

$$f^{s}(s^{A}_{B}, A, B) = \frac{(n+m)\left(\left|\{x \in A : x \leq s^{A}_{B}\}\right| + \left|\{x \in B : s^{A}_{B} \leq x\}\right|\right) + \left|\{x \in A : s^{A}_{B} \leq x\}\right| + \left|\{x \in B : x \leq s^{A}_{B}\}\right|}{(n+m)(n+m+1)}$$
(4)

The first term, which is multiplied with (n + m), counts the number of alternatives in A that outrank the separating profile and the number of alternatives in B that are outranked by it. If all the alternatives in A and B are counted, then the separating profile is not strictly preferred to any of the alternatives in A, while none of the alternatives in B are strictly preferred to it. In this case, the separating profile may be said to have been placed between the two sets of alternatives. However, we may have certain alternatives from both sets that are indifferent to the separating profile. In this case the separating profile may not be considered to properly separate A and B. For this reason we have added the second term from Equation (4), which counts the number of alternatives from A that are not outranked by s_B^A , and the number of alternatives from B that do not outrank s_B^A . If this term is also maximized then all the alternatives from A will be strictly preferred to the separating profile, while all the alternatives from B will be strictly preferred by it.

3 Algorithmic Approaches to Determine the Profiles

Several approaches to constructing the presented profiles may be considered, from very simple ones to others that are more complex. Some of them are independent of the preference model that is used in order to reflect the perspective of the DM over the set of alternatives, while others are tailored for a particular type of outranking relation. We will consider in the case of the latter, the outranking relation from [2], although the approaches that we will present may easily be adapted to other outranking relations.

For the selected relations, the "at least as good as" comparisons are characterized for all pairs of alternatives x and y and for all criteria $i \in F$ by:

$$C_i(x,y) = \begin{cases} 1 \text{ if } y_i < x_i + q_i ;\\ -1 \text{ if } y_i \ge x_i + p_i ;\\ 0 \text{ otherwise }, \end{cases}$$
(5)

where $0 \leq q_i$ (resp. $p_i \geq q_i$) is a constant indifference (resp. preference) threshold associated with the *i*th criterion. A weight $w_i > 0$ is associated with each criterion *i*, and the overall concordance index C(x, y) is defined as the weighted sum of the marginal concordances. A veto threshold v_i for each criterion i is also introduced in order to invalidate the outranking in case a very large difference of evaluations on at least one criterion is detected in favour of the overall less preferred alternative. Consequently, an alternative x outranks an alternative y $(x \, S \, y)$ iff C(x, y) > 0 and $y_i - x_i < v_i \ \forall i \in F$.

3.1 Exact Approaches

One of the simplest approaches is to **select** an existing alternative based on how well it performs with respect to the considered fitness measures. For instance a central profile is selected as follows:

$$c_A = \operatorname*{arg\,max}_{x \in A} f^c(x, A). \tag{6}$$

Not only will such an approach be very fast, but it will also give the DM a result with which he is familiar, as the profiles are real alternatives.

Nevertheless, profiles that are selected from the existing alternatives may not always be of good quality, considering the fitness measures we have proposed. This may easily be imagined for sets containing very contrasting alternatives.

Another simple and quick approach for building these profiles is to **construct** them directly from the evaluations of the alternatives. In the case of a central profile we may consider a simple mean operator as follows:

$$c_{Ai} = \frac{1}{n} \sum_{x \in A} x_i, \forall i \in F.$$

$$\tag{7}$$

This approach is only suited when the criteria are defined on quantitative scales, however we may use a median operator when confronted with ordinal scales.

In the case of bounding profiles, since the upper bounding profile should mainly outrank the alternatives in A, while the lower bounding profile should mainly be outranked by them, we may use the max and min operators:

$$b_{A_i}^+ = \max_{x \in A} x_i, \forall i \in F, \qquad b_{A_i}^- = \min_{x \in A} x_i, \forall i \in F.$$
(8)

A separating profile may be given as the mean between the central profiles of the two sets:

$$s_{B_{i}}^{A} = \frac{1}{2} \Big(\frac{1}{n} \sum_{x \in A} x_{i} + \frac{1}{m} \sum_{x \in B} x_{i} \Big), \forall i \in F.$$
(9)

While these approaches for building central, bounding or separating profiles are simple and fast, nothing guarantees that they will find a good result with respect to the fitness measures defined in Section 2.

A third approach is to use **mathematical programs** that model the outranking relations between alternatives to extract the central, bounding or separating profiles which are optimal with respect to the fitness measures defined in Section 2. We have considered an extension of the work of [4], which may be used to model the outranking relation presented earlier in this section, in order to determine the optimal profiles in an exact way. However, due to complexity issues, such an approach quickly becomes impractical when considering larger sets of alternatives. As we will consider such cases for the empirical validation of the presented algorithmic approaches in Section 4, we do not elaborate further on the topic of using a mathematical program.

3.2 Meta-heuristic Approach

An alternative to finding an optimal central, bounding or separating profile is to perform a trade-off between the quality of the profile and the time required by the approach in order to find it. Hence, we may use meta-heuristic approaches [10], which find results that are close to the optimal one in a fraction of the time required by exact approaches.

In our case, any single-solution meta-heuristic may be used. We present the outline of these approaches below [10]:

Algorithm 1. Single-solution meta-heuristic
Input: Initial solution s_0 .
1: $t = 0;$
2: while not Stopping criterion satisfied do
3: $N(s_t) = \text{GENERATE}(s_t); /* \text{Generate candidate solutions from } s_t */$
4: $s_{t+1} = \text{SELECT}(N(s_t));$ /* Select a solution to replace the current one */
5: t = t + 1;
Output: Best solution found.

The initial solution may either be constructed randomly, or may be guided towards a good solution. In our case we will be using the first approach of selecting an existing alternative that maximizes the considered fitness measure.

The neighbours of the current solution will be those that contain an evaluation change on only one criterion. This change will be either the smallest increase or the smallest decrease of the evaluation, which would change the way in which the profile compares on a particular criterion to the alternatives that it tries to describe. We motivate this by the desire to be able to explore the search space from one neighbouring solution to the next, without performing large changes to a profile, which may lead us to miss potentially better intermediate solutions.

The selection of the new solution generally depends on the actual type of meta-heuristic used. Nevertheless, in many cases the neighbouring solutions are evaluated based on the fitness measure and then a selection procedure is applied. However, it may be the case that assessing the fitness of all neighbouring solutions, or even constructing them, will increase the execution time of the approach. In such cases, certain heuristics may be used to assess the quality of each change on the current solution. We will propose in what follows different heuristic measures for each of the three types of profiles that have been defined in this paper. The outranking relation which we use here is the one defined in the beginning of this section. Note that similar heuristics can be given for other definitions of the outranking relation.

We begin with the heuristic for increasing the evaluation of a central profile on a particular criterion $i \in F$, considering the alternatives in set A:

 $h^{c}(c_{A},i) = \left| \{ x \in A : x_{i} - c_{Ai} > q_{i} \land c_{A} \not \!\!\! x \} \right| - \left| \{ x \in A : x_{i} - c_{Ai} < -q_{i} \land c_{A} \not \!\!\! x \} \right|$ (10)

Since the central profile should be indifferent to the alternatives in A, the heuristic in Equation (10) may be seen as a voting procedure where each alternative in A votes in favour of increasing the evaluation of c_A on criterion i,

in disfavour, or refrains from voting. This is reflected by the two terms in this equation. The first term counts the number of alternatives which have an evaluation higher than that of c_A by more than the q_i threshold. This means that those alternatives are not considered indifferent to c_A on criterion *i*. Moreover, those alternatives are preferred to it, therefore, from their perspective, the evaluation of c_A should be increased in order for them to become indifferent. The second term counts in a similar way the alternatives from whose perspective the evaluation of c_A on criterion *i* should be decreased. The alternatives which are already indifferent with c_A on criterion *i* do not require an increase or decrease in the evaluation of c_A furthermore, the alternatives in *A* that are already overall indifferent to c_A do not take part in this process, even if their evaluations on criterion *i* are not indifferent to that of c_A , as this would not increase the fitness of the central profile. The heuristic is valued in the [-n, n] interval.

The heuristic of decreasing the evaluation of c_A on criterion *i* is $-h^c(c_A, i)$.

In Figure 1 we illustrate the way in which the heuristic works, considering a set containing only four alternatives.



Fig. 1. Detailing the heuristic for changing c_A for a set A of 4 alternatives

In this example we consider a set $A = \{x, y, z, t\}$ of four alternatives and their central profile c_A . We consider that none of these alternatives are at this point overall indifferent to c_A , therefore they all take part in the voting process. It is evident that the evaluation of c_A should be increased, as two alternatives from A are in favour of this change, one is against and another refrains from voting, therefore giving a positive value to the heuristic measure. However, we would only add to the evaluation of c_A the smallest amount which changes at least one of the comparisons between it and the alternatives in A. The first alternative, x, would require c_A to be increased by an amount that brings the first dotted line below the evaluation of x on i just above it. This amount is $x_i - c_{A_i} - p_i + \epsilon$, where $\epsilon > 0$ and $\epsilon \ll 1$, as in this case x would become only weakly preferred

to c_{A_i} . However, this amount can be seen to be larger than the amounts that would be required in order for the other alternatives compare differently to c_A , therefore we will not increase c_{A_i} by this amount. The use of ϵ is necessary following the definition of the outranking relation. The smallest amount that would impact the way in which at least one alternative compares to c_A on i is equal to $y_i - c_{A_i} - q_i$, which would make y become indifferent to c_A on criterion i, while all the other alternatives will remain in the same state as before. Therefore the increase of c_{A_i} would be this amount.

Having a positive fitness value for the heuristic in Equation (10) does not imply that we would increase its evaluation. All the operations of both increasing and decreasing the evaluations of c_A on all criteria, characterised through the described heuristic measure, are used in the meta-heuristic approach.

We continue with the heuristic functions for increasing the evaluations of the bounding profiles on a particular criterion in Equations (11) and (12).

$$h^{b+}(b_A^+, i) = n \cdot \left| \{ x \in A : x_i - b_A^+ > q_i \land b_A^+ > x_i \} \right| - \left| \{ x \in A : x_i - b_A^+ < -q_i \land x > b_A^+ \} \right| \quad (11)$$

$$h^{b-}(b_{A}^{-},i) = \left| \left\{ x \in A : x_{i} - b_{Ai}^{-} > q_{i} \wedge b_{A}^{-} \, \$ \, x \right\} \right| - n \cdot \left| \left\{ x \in A : x_{i} - b_{Ai}^{-} < -q_{i} \wedge x \, \$ \, b_{A}^{-} \right\} \right|$$
(12)

We find that these heuristics are defined similarly to the one for a central profile. The first term from both counts the number of alternatives that do not outrank each profile but have an evaluation that is above that of the profile by more than the indifference threshold. In this case the evaluation of the profile should be increased so that it would outrank the considered alternatives on criterion i. Similarly, in the second term the alternatives that are not outranked by the bounding profiles and that have their evaluations lower by more than the indifference threshold require the evaluations of the profiles to be decreased. The two terms are weighted so that one of them dominates the other, as is the case with the fitness measures for these profiles.

Finally, we present the heuristic for increasing the evaluation of a separating profile, considering the two sets A and B:

$$h^{s}(s_{B}^{A}, i) = (n+m) \left(\left| \left\{ x \in B : x_{i} - s_{B_{i}}^{A} > q_{i} \land s_{B}^{A} \, \$ \, x \right\} \right| - \left| \left\{ x \in A : x_{i} - s_{B_{i}}^{A} < -q_{i} \land x \, \$ \, s_{B}^{A} \right\} \right| \right) + \left| \left\{ x \in B : x_{i} - s_{B_{i}}^{A} > -q_{i} \land x \, \$ \, s_{B}^{A} \right\} \right| - \left| \left\{ x \in A : x_{i} - s_{B_{i}}^{A} < q_{i} \land s_{B}^{A} \, \$ \, x \right\} \right|$$
(13)

The first term counts the alternatives from B that require the evaluation of s_B^A on i to be increased in order for it to outrank them, while the second term counts the alternatives from A that require that this evaluation is lowered in order for them to outrank the separating profile. These terms are weighted, as they account for the most important part of the definition of a separating profile. The following two terms account for the alternatives in B that require an increase in the evaluation of s_B^A , and those from A that require a decrease.

4 Empirical Validation

In order to be able to compare the performance of the proposed approaches for extracting each type of profile, we have generated a large number of problem instances containing one or two sets of alternatives. We have fixed the size of these sets of alternatives to 50, making them very difficult for a DM to analyse directly.

4.1 Constructing the Benchmarks

The alternatives are defined on a number of 11 criteria which are valued on ratio scales in the interval [0,1]. This number has been chosen in order for the alternatives to resemble those from real problems that are considered to be difficult, but also allowing us to construct very diverse ones. In order to model a wide range of potential problems, we also generate the evaluations of the alternatives in each set so that they are increasingly contrasting. A total of ten generators are used, which we denote alphabetically from \mathcal{A} to \mathcal{J} . While the first builds each alternative using a normal distribution centred at the median level on every criterion, the following four randomly pick for each alternative normal distributions that are increasingly spaced apart. The following generators are the same as the first five except that very good and very bad performance evaluations are additionally inserted. For each alternative, two distinct criteria are randomly picked and with a 50% probability the evaluation on the first criterion is maximized, while with the same probability the evaluation on the second criterion is minimized. Using each generator we have built 5 problem instances.

The perspective of a fictive DM on these sets of alternatives is modelled using the outranking relation from [2]. The criteria have been given equal importance weights as we are not dealing with real decision problems, but also due to the fact that by reducing the significance of certain criteria in favour of others we reduce the impact that they would have on the way in which the alternatives compare to each other. By maintaining the criteria importance weight the same for all of the criteria, we are assuring that the benchmarks have the highest diversity in their structures as possible. The discrimination thresholds are selected so that evaluations that are generated using the same normal distribution are in a high percentage indifferent. Only one veto threshold is used, which is set to three quarters of the value range, making veto situations appear very rarely inside the instances constructed using the first five generators.

4.2 Results

For each of the 50 problem instances that we have generated, we have constructed the central, bounding and separating profiles using the three approaches proposed in this paper. The approaches of selecting existing alternatives and that of constructing them from the evaluations of the alternatives in the sets have been executed only once on each benchmark, as they are deterministic.

In the case of the meta-heuristic approaches, we have selected a simulated annealing implementation [3]. The initialisation step is given the solution of the first of the previous approaches, while the cooling rate is fixed so that the algorithm will run at most for one minute. This limit has been set in order to simulate real-life conditions where the approaches of constructing these profile

need to quickly output good results. Furthermore, a strategy using restarts had been additionally applied. This approach has been executed 50 times over each benchmark in order for the results with respect to the average fitness of the profiles to be significant.

The average fitness of the three types of profiles, as well as the standard deviations, where relevant, are presented for each of the ten types of benchmarks in Figure 2.



Fig. 2. Average fitness of the central, bounding and delimiting profiles

Certain conclusions may be drawn for the results of finding any type of profile. First of all, we notice that the approaches find profiles that are less fit with respect to the considered fitness measures as we tackle problems instances that contain increasingly contrasting alternatives. This is seen through the decrease in fitness from the first type of benchmarks up to the fifth, as well as from the sixth and up to the last. The two sets of benchmarks resemble strongly each other, except that in the case of the second set we have added large performance gains or losses for certain alternatives in the sets.

Secondly, we may notice that the approaches of building the profiles using simple operation on the evaluations of the alternatives in each set generally perform poorer than all the rest. A few exceptions occur when constructing bounding profiles, where the proposed approach is always able to build an upper bounding profile that outranks all the alternatives in the set and a lower bounding profile that is outranked by them. Any fitness results that are above these highlight the fact that we have constructed bounding profiles that come closer to the alternatives in the set, i.e. indifferent to them. The meta-heuristic approaches improve on the results given by the approach of selecting an existing alternative from the dataset. The largest improvements may be seen to occur for the approach of constructing a central profile, however this is due to the nature of the fitness measure. Nevertheless, in this case we are able to improve the results of the approach of selecting an existing alternative by as much as 10%.

For the other types of profiles the fitness measures model two objectives, the first dominating the other, and thus improvements over the less important objective are less visible. Nevertheless, when the first objective is maximized the second one becomes also very important. This is especially the case for the bounding profiles, which we additionally want to become indifferent to as many of the alternatives in the set as possible. We find that in this case the first approach is already performing quite well for the first types of benchmarks, and the meta-heuristic is not able to improve on its results.

Finally, in the case of the separating profiles, the meta-heuristic approach performs quite well, for certain benchmark type being able to find separating profiles that are strictly preferred by the alternatives in the first set and strictly preferred to the alternatives in the second set.

5 Conclusions and Perspectives

In this paper we have proposed three types of profiles that may be used in order to describe one or several sets of alternatives. Two of these profiles, the central and separating profiles, have been extended from the context of the problem of sorting, while the bounding profiles are new. Through them we are able to reduce one or two sets of alternatives to a condensed representation, which would aid a DM in understanding and dealing with these sets as a whole, especially when we are dealing with a large number of alternatives inside them. The definitions of these profiles make their use very intuitive.

For each of the three types of profiles we have presented three approaches for constructing them, which we have tested over a large number of benchmarks holding various difficulties. The results show that in most cases the approach of selecting an existing alternative performs quite well, however using a metaheuristic we are able to find even better results. Furthermore, the approaches of constructing the profiles using simple operations over the evaluations of the alternatives in the sets in general perform worse than the others.

Although we have considered the use of a mathematical program in order to find the optimal central, bounding or separating profiles, due to the size of the sets of alternatives such an approach became highly impractical.

We would like to consider in the future extending these profiles for other definitions of outranking relations as well as additionally considering the credibility degrees that are normally associated with them.

We envision the use of these profiles mainly in the problem of clustering in MCDA [5]. As clustering may be seen as an exploratory data analysis technique, being able describe clustering results over large sets of alternatives, using a considerably smaller set of central, bounding or separating profiles, would greatly

enhance the exploration and understanding of the original dataset. Furthermore, these profiles could also be used in conjunction with the problems of choice and ranking, provided that large groups alternatives are generated as results to these problems. Finally, being able to summarize the information given by a set of alternatives may additionally aid in a process of eliciting the preferences of a DM over large sets of alternatives. We will explore these topics in the future.

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