Introduction

Multiple Criteria Decision Aid

• aims at modelling the preferences of decision-makers;
• aids them in reaching certain decisions;

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Price ↓</th>
<th>Acceleration ↓</th>
<th>Safety ↑</th>
<th>···</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car 1</td>
<td>18,342</td>
<td>30.7s</td>
<td>good</td>
<td>···</td>
</tr>
<tr>
<td>Car 2</td>
<td>15,335</td>
<td>30.2s</td>
<td>medium</td>
<td>···</td>
</tr>
<tr>
<td>Car 3</td>
<td>16,973</td>
<td>29s</td>
<td>v.good</td>
<td>···</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
<td>...</td>
</tr>
</tbody>
</table>
Introduction

Modelling preferences

Value functions (U)
- each alternative receives a **score**;
- \( U(x) \) = aggregated criteria evaluations of \( x \);
- **trade-offs** between criteria;

Outranking relations (S)
- alternatives are compared pair-wisely:
  1) is \( x \) **at least as good as** \( y \) on a weighted majority of criteria?
  2) is \( x \) **not much worse** than \( y \) on any criterion?
- similar to **voting**;

Preferential situations
\[
\begin{align*}
U(x) &= U(y) & \text{Indifference (I)} & & x S y \land y S x \\
U(x) &> U(y) & \text{Strict preference (P)} & & x S y \land y S x \\
U(x) &\geq U(y) & \text{Weak preference (Q)} & & x S y \\
& & \text{Incomparability (R)} & & x S y \land y S x
\end{align*}
\]

Main typologies of problems
Main typologies of problems

Best choice

Ranking

Sorting

Clustering

Alternatives
Introduction

Defining the profiles

Finding the profiles

Results

Conclusions and perspectives

Decision Aiding process

Motivation

Set of alternatives

Decision Maker
Motivation

Defining the profiles

Setting
- a set of alternatives \( A \);
- a set of criteria \( F \);
- \( x_i \) evaluation of \( x \in A \) on \( i \in F \);
- outranking relation \( S \rightarrow \) indifference relation \( I \), strict preference relation \( P \) and incomparability \( R \);

Central profile \((c_A)\)
- indifferent to the alternatives in \( A \);
- \( f(c_A) = |\{x \in A: x \perp c_A\}| \);
- may be used to replace \( A \);
- useful for representing sets of indifferent alternatives.
Bounding profiles \((b_{A}^{+}, b_{A}^{-})\)

1. \(b_{A}^{+}\) is above \(A\)
   (not strictly preferred by \(\forall x \in A\));
   \(b_{A}^{-}\) is below \(A\)
   (not strictly preferred to \(\forall x \in A\));
2. \(b_{A}^{+}, b_{A}^{-}\) are close to \(A\)
   (indifferent to as many \(x \in A\));
\[\diamond \ f(b_{A}^{+}) = |A| \cdot \{x \in A: b_{A}^{+} S x\}
+ \{x \in A: x S b_{A}^{+}\};\]
\[\diamond \ f(b_{A}^{-}) = |A| \cdot \{x \in A: x S b_{A}^{-}\}
+ \{x \in A: b_{A}^{-} S x\};\]
- extend a central profile;
- useful for representing sets of less indifferent alternatives.

Separating profile \((s_{A}^{A}\))

1. \(s_{A}^{A}\) is between \(A\) and \(B\)
   (not strictly preferred to \(\forall x \in A\) and
   not strictly preferred by \(\forall x \in B\));
2. \(s_{A}^{A}\) is separated from \(A\) and \(B\)
   (not indifferent to as many \(x \in A \cup B\));
\[\diamond \ f(s_{A}^{A}) = (n + m) \cdot \{x \in A: x S s_{A}^{A}\} +
(n + m) \cdot \{x \in B: s_{A}^{A} S x\} +
\{x \in A: s_{A}^{A} S x\} +
\{x \in B: x S s_{A}^{A}\};\]
- useful for delimiting two sets of alternatives that are ordered;

Finding the profiles

Exact approaches

Selecting
- an existing alternative from \(A\) (or \(B\)) that maximizes \(f\);

Building
- a fictitious alternative from the evaluations of \(x \in A\) (or \(B\));

\[c_{A_{i}} = \frac{1}{n} \sum_{x \in A} x_{i}, \quad b_{A_{i}}^{+} = \max_{x \in A} x_{i}, \quad b_{A_{i}}^{-} = \min_{x \in A} x_{i}, \quad s_{B_{i}}^{A} = \frac{1}{2} (c_{A_{i}} + c_{B_{i}});\]
- using a linear program that models the outranking relation \(S\)
  ([Bisdorff, Meyer, Roubens 07],[Bisdorff 12]) between the profiles and
  the alternatives in \(A\) and \(B\).

* with only one veto threshold
Approximative approach

Meta-heuristic

- **single solution meta-heuristic:**
  - start from an initial solution;
  - iteratively change it until a stop criterion is met;

- **tested simulated annealing:**
  - ability to escape local optima;
  - relatively easy to tune (cooling schedule);
  - may use restarts;

- used the outranking relation $S$ from [Bisdorff, Meyer, Roubens 07] with only one veto threshold;
  - proposed a **heuristic** for the algorithm.

Heuristic

Experiments description:

- constructed a series of 50 benchmarks:
  - 50 alternatives;
  - 11 criteria;
  - $[0, 1]$ ratio scales;
  - 10 classes of difficulty ($A - J$);

- considered a fictive DM:
  - outranking relation $S$ from [Bisdorff, Meyer, Roubens 07] with only one veto threshold;
  - equally significant criteria;
  - indifference, preference and veto thresholds;
  - median cut ($\lambda = 0.5$);

- executed all the approaches (except linear programs $> 60$ min) (50 executions over each benchmark, 10 seconds each);
- compared results w.r.t. the fitness measures.
Results

Conclusions and perspectives

Conclusions

- **selecting** an alternative is generally better than **constructing** one from mean, max or min evaluations;
- meta-heuristic provides **significant improvements** over exact approaches for central profiles (over 5% even when using the credibility of the indifference relation);
- improvements for bounding and separating profiles are not so visible (modelling the two objectives);
- using **min** and **max** evaluations for bounding profiles maximizes the first set of objectives → should only model the second (which brings the profiles closer to the alternatives);
Perspectives

- further investigation into bounding and separating profiles and the representation of their fitness;
- finding the optimum result for each benchmark;
- inclusion of the veto in the heuristic;
- easy extension of using the weights in the heuristic;
- application for describing clustering results.
Conclusions and perspectives

Introduction
Defining the profiles
Finding the profiles
Results
Conclusions and perspectives

Perspectives

Decision Maker

\[ \{a_1, \ldots, a_m\} = c_a \]
\[ \{b_1, \ldots, b_m\} = c_b \]
\[ \{c_1, \ldots, c_m\} = c_c \]

Decision Maker

\[ \{a_1, \ldots, a_m\} = c_a \]
\[ \{b_1, \ldots, b_m\} = c_b \]
\[ \{c_1, \ldots, c_m\} = c_c \]