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Descriptive profiles for sets of alternatives in MCDA

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Introduction

Introduction

Multiple Criteria Decision Aid

- aims at modelling the **preferences** of decision-makers;
- **aids** them in reaching certain **decisions**;

Alternatives	Criteria			
	Price ↓	Acceleration ↓	Safety ↑	···
Car 1	**18,342**	**30.7s**	good	···
Car 2	**15,335**	30.2s	**medium**	···
Car 3	16,973	**29s**	**v.good**	···
⋮	⋮	⋮	⋮	

Modelling preferences

Value functions (U)

- each alternative receives a **score**;
- $U(x)$ = aggregated criteria evaluations of x;
- **trade-offs** between criteria;

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  1) is x **at least as good as** y on a **weighted majority** of criteria?

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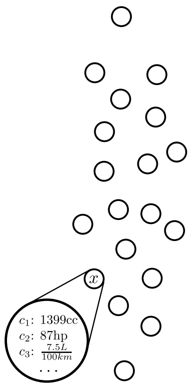
Preferential situations

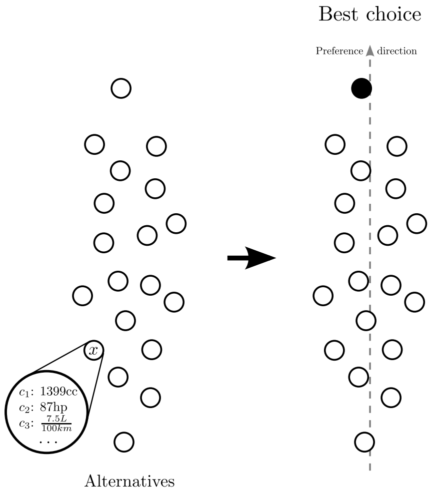
$U(x) = U(y)$	Indifference (I)	$x \, S \, y \wedge y \, S \, x$
$U(x) > U(y)$	Strict preference (P)	$x \, S \, y \wedge y \not S \, x$
$U(x) \geqslant U(y)$	Weak preference (Q)	$x \, S \, y$
	Incomparability (R)	$x \not S \, y \wedge y \not S \, x$

Main typologies of problems



Alternatives

Main typologies of problems



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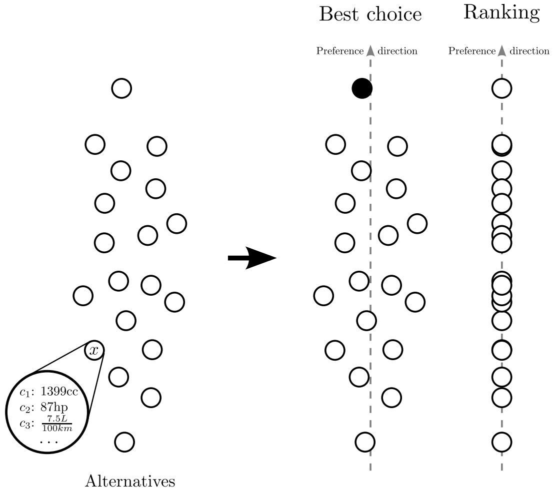

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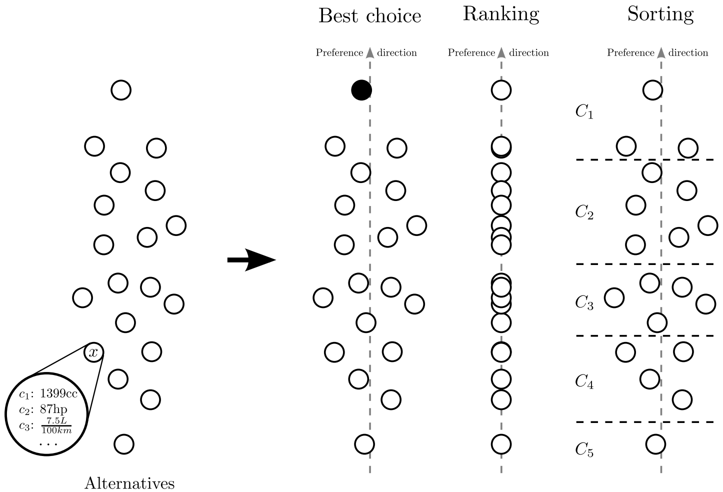

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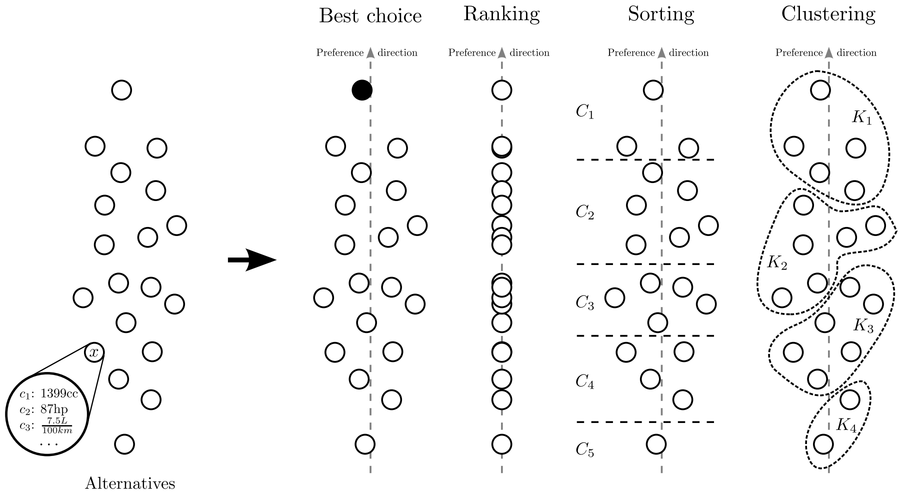
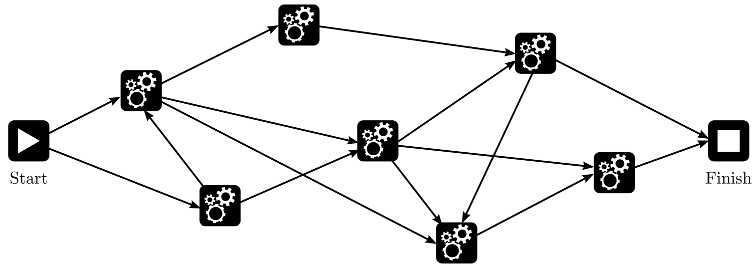


Decision Aiding process



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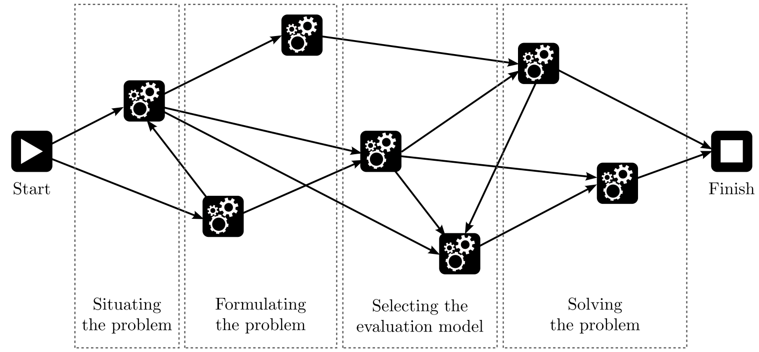

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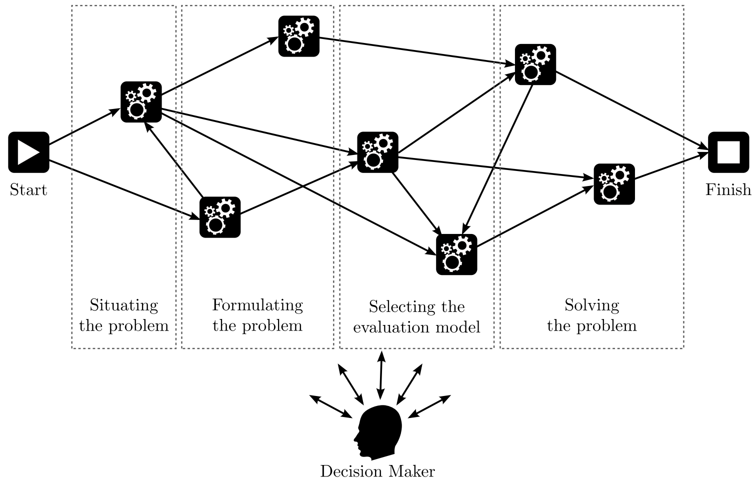
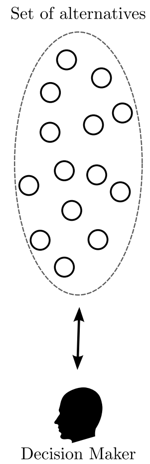
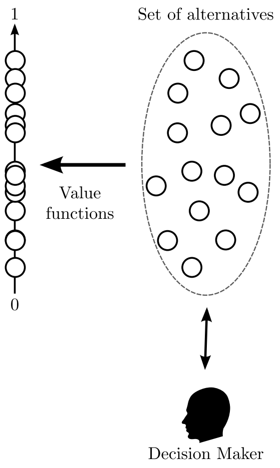


Motivation



Motivation

- maximum;
- mean;
- standard deviation;
- histogram;
- minimum.



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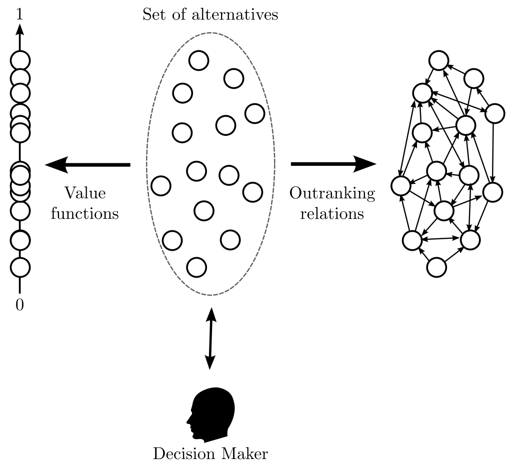


- top/best alternative;
- central/equivalent alternative;
- bottom/worst alternative;
- # alternatives indifferent to top alternative;
- # alternatives indifferent to central alternative;
- # alternatives indifferent to bottom alternative.

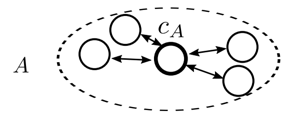
Defining the profiles

Setting

- a set of alternatives A;
- a set of criteria F;
- x_i evaluation of $x \in A$ on $i \in F$;
- outranking relation S $\rightarrow$ indifference relation I, strict preference relation P and incomparability R;

Central profile (c_A)

- **indifferent** to the alternatives in A;
- ◇ $f(c_A) = \left|\{x \in A:\ x \,\mathrm{I}\, c_A\}\right|$;
- may be used to **replace** A;
- useful for representing sets of indifferent alternatives.



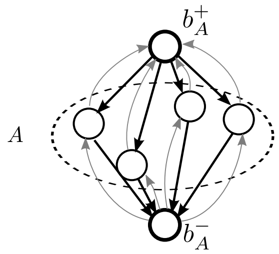
Bounding profiles (b_A^+, b_A^-)

1. b_A^+ is above A
   (**not strictly preferred by** $\forall x \in A$);
   b_A^- is below A
   (**not strictly preferred to** $\forall x \in A$);
2. b_A^+, b_A^- are close to A (**indifferent to** as many $x \in A$);

◇ $f(b_A^+) = |A| \cdot \left|\{x \in A\colon\ b_A^+ \, S \, x\}\right| + \left|\{x \in A\colon\ x \, S \, b_A^+\}\right|$;

◇ $f(b_A^-) = |A| \cdot \left|\{x \in A\colon\ x \, S \, b_A^-\}\right| + \left|\{x \in A\colon\ b_A^- \, S \, x\}\right|$;

- **extend** a central profile;
- useful for representing sets of less indifferent alternatives.

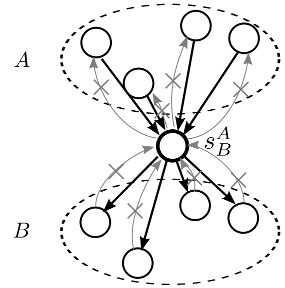


Separating profile (s_B^A)

1. s_B^A is between A and B
   (**not strictly preferred to** $\forall x \in A$ and **not strictly preferred by** $\forall x \in B$);
2. s_B^A is separated from A and B
   (**not indifferent to** as many $x \in A \cup B$);

◇ $f(s_B^A) = (n+m) \cdot \left|\{x \in A\colon\ x \, S \, s_B^A\}\right| + (n+m) \cdot \left|\{x \in B\colon\ s_B^A \, S \, x\}\right| + \left|\{x \in A\colon\ s_B^A \, \not S \, x\}\right| + \left|\{x \in B\colon\ x \, \not S \, s_B^A\}\right|$;

- useful for delimiting two sets of alternatives that are ordered;



Finding the profiles

Exact approaches

Selecting

- an existing alternative from A (or B) that maximizes f;

Building

- a fictitious alternative from the evaluations of $x \in A$ (or B);

$$c_{A\,i} = \frac{1}{n} \sum_{x \in A} x_i \qquad b_{A\,i}^+ = \max_{x \in A} x_i \qquad b_{A\,i}^- = \min_{x \in A} x_i \qquad s_{B\,i}^A = \frac{1}{2}\left(c_{A\,i} + c_{B\,i}\right)$$

- using a linear program that models the outranking relation S ([Bisdorff, Meyer, Roubens 07]*,[Bisdorff 12]) between the profiles and the alternatives in A and B.

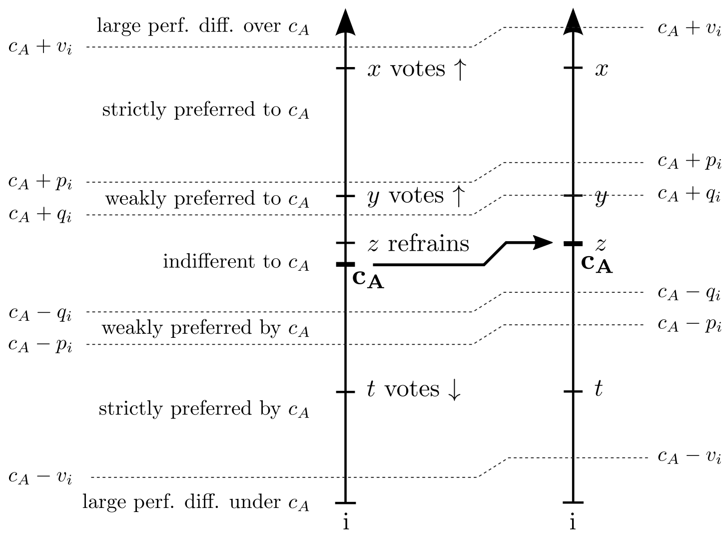
* with only one veto threshold

Approximative approach

Meta-heuristic

- single **solution meta-heuristic**:
  - start from an initial solution;
  - iteratively change it until a stop criterion is met;
- tested **simulated annealing**:
  - ability to **escape** local optima;
  - relatively easy to tune (cooling schedule);
  - may use restarts;
- used the outranking relation S from [Bisdorff, Meyer, Roubens 07] with only one veto threshold;
  - proposed a **heuristic** for the algorithm.

Heuristic

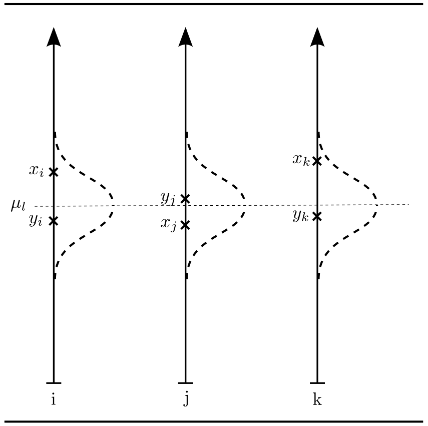


Results

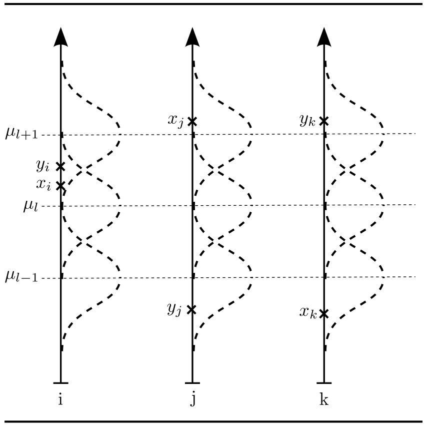
Experiments description:

- constructed a series of 50 benchmarks:
  - 50 alternatives;
  - 11 criteria;
  - $[0, 1]$ ratio scales;
  - 10 classes of difficulty ($\mathcal{A}$ - $\mathcal{J}$);
- considered a fictive DM:
  - outranking relation S from [Bisdorff, Meyer, Roubens 07] with only one veto threshold;
  - equally significant criteria;
  - indifference, preference and veto thresholds;
  - median cut ($\lambda = 0.5$);
- executed all the approaches (except linear programs > 60 min) (50 executions over each benchmark, 10 seconds each);
- compared results w.r.t. the fitness measures.

Benchmark construction

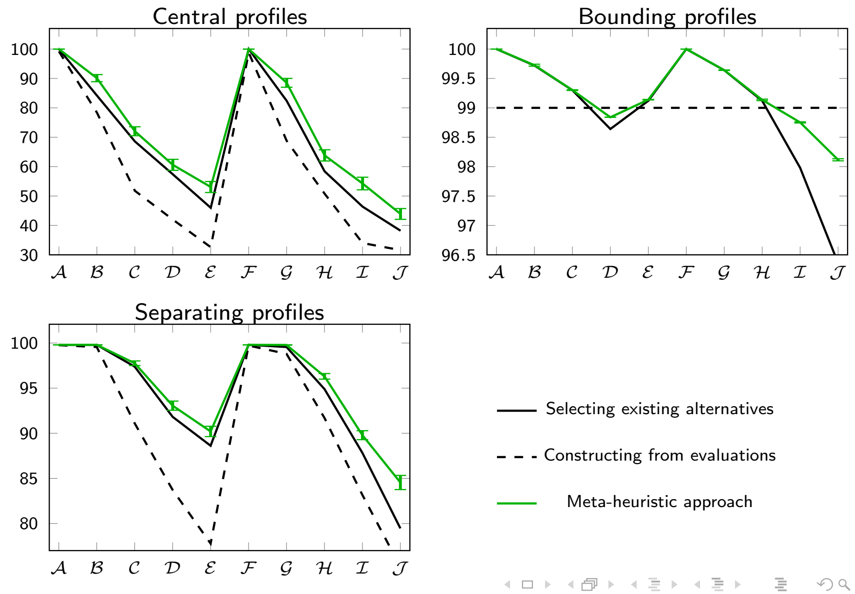


Generator $\mathcal{A}$;

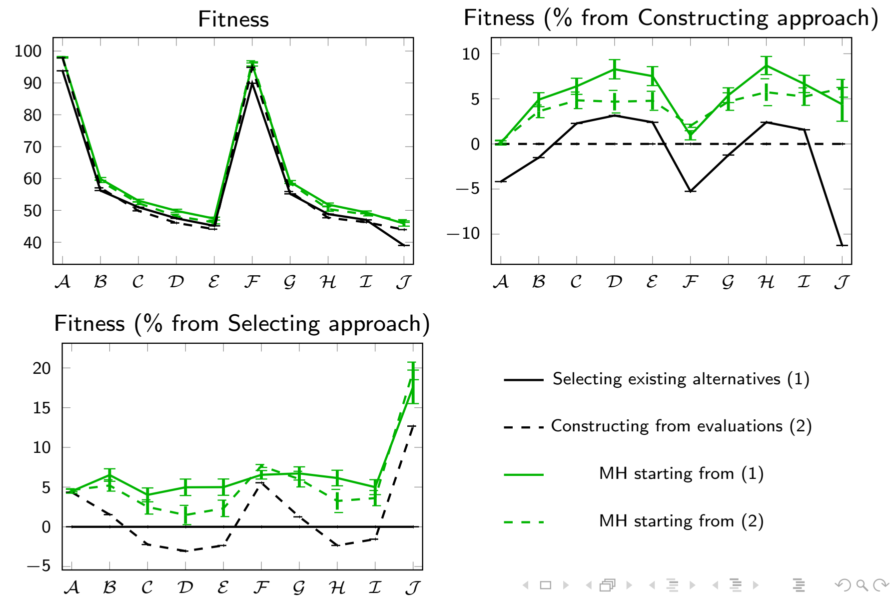


Generator $\mathcal{B}$;

Results for all profiles



Results for central profiles using a valued indifference relation



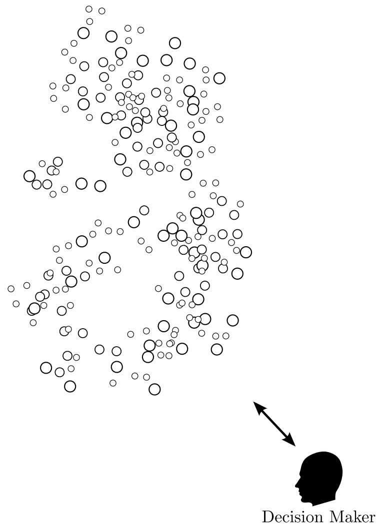
Conclusions

- **selecting** an alternative is generally better than **constructing** one from mean, max or min evaluations;
- meta-heuristic provides **significant improvements** over exact approaches for central profiles (over 5% even when using the credibility of the indifference relation);
- improvements for bounding and separating profiles are not so visible (modelling the two objectives);
- using **min** and **max** evaluations for bounding profiles maximizes the first set of objectives → should only model the second (which brings the profiles closer to the alternatives);

Perspectives

- further investigation into bounding and separating profiles and the representation of their fitness;
- finding the optimum result for each benchmark;
- inclusion of the veto in the heuristic;
- easy extension of using the weights in the heuristic;
- application for describing clustering results.

Perspectives



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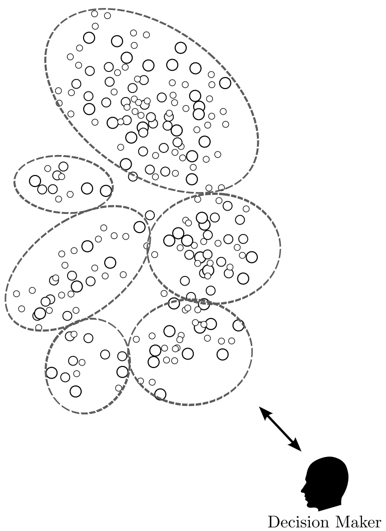

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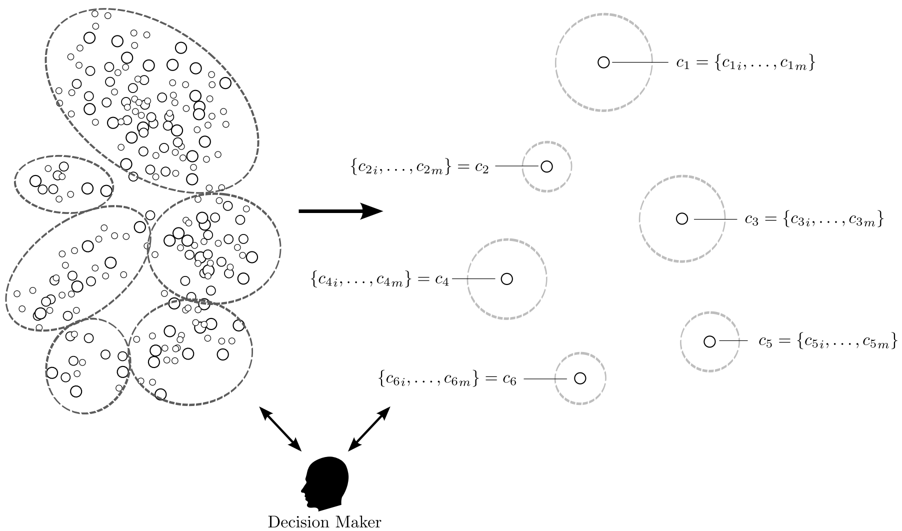

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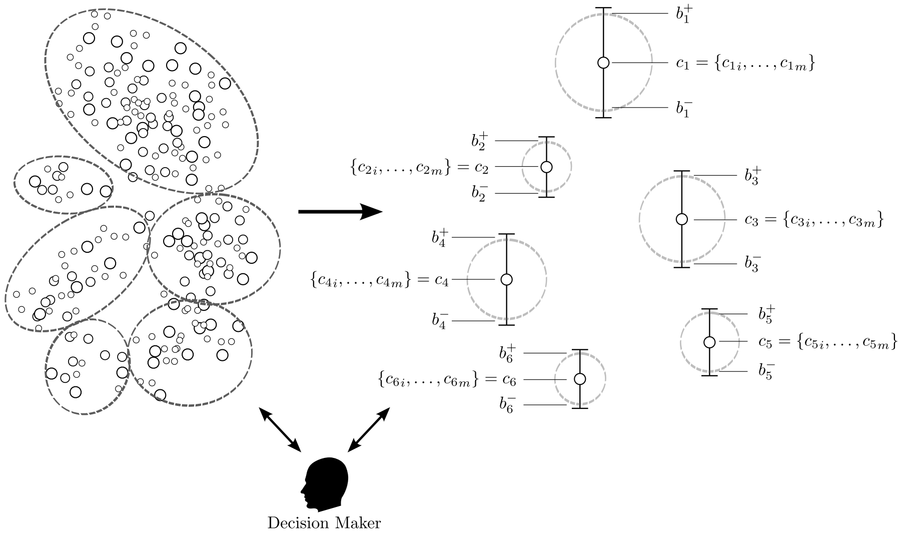

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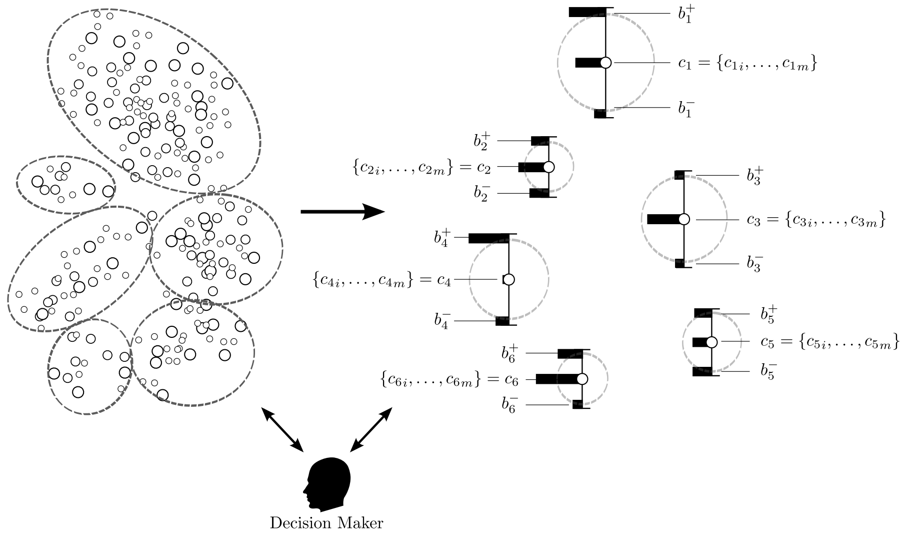
Perspectives



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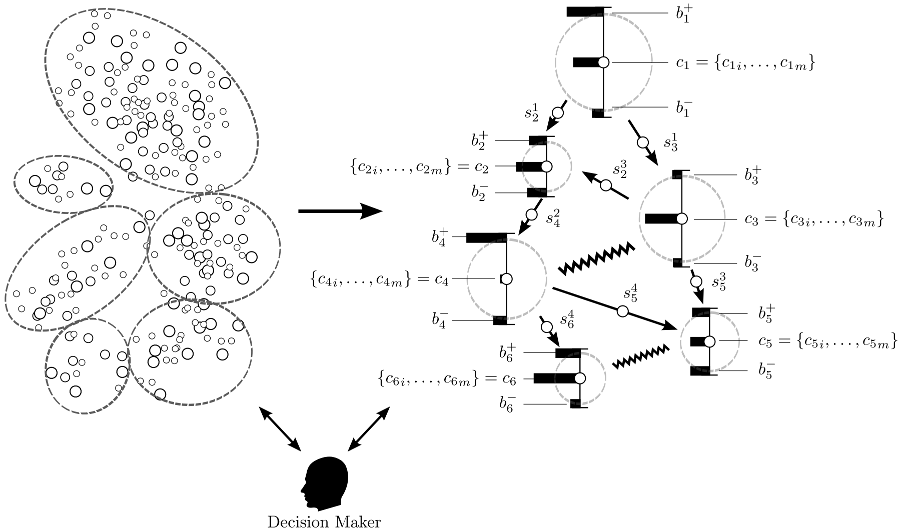

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