Motivation	Valued Outranking Digraphs 0 00 0	Random Performance Tableaux o o o	Random Outranking Digraphs 0 00 00	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux o oo o	Random Outranking Digraphs O OO OO	Conclusion
	Rando	m Outranking D	ligraphs		1.1 1.2 1.3	Valued Outranking The outranking si The outranking in The bipolar valued	tuation		
	Raymond Bisdorff (and Bernard Roy) University of Luxembourg, FSTC/CSC					Random Performa Reference model random performar random thresholds			
		Leuven, January, 200	9			Random Outranki Definition Link densities Connectivity			
Motivation	Valued Outranking Digraphs	Random Performance Tableaux O	← □ > < ♂ > < ≥ > < ≥ > Random Outranking Digraphs Oc	≥ · 카직 (> Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux O	C □ > < ● > < ≥ > < ≥ > Random Outranking Digraphs 0 0	ই ৩৭৫ Conclusion

Motivation

- Provide instances of genuine valued outranking digraphs (VODs) for MCDA method debugging
- Discover VOD's specific structural characteristics
- Comparison with other kinds of random valued digraphs
- Mathematical characterisation of VODs

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Valued Outranking Digraphs	Random Performance Tableaux 0 00 0	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux 0 00 0	Random Outranking Digraphs 0 00 00	Conclusion		
	Motivation					Motivation				
		tranking digraphs		 Provide instances of genuine valued outranking digraph (VODs) for MCDA method debugging 						
Discover VOD's sp	pecific structural char	acteristics		•	Discover VOD's s	pecific structural cha	racteristics			
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	 Provide instances (VODs) for MCD/ Discover VOD's sp Comparison with or 	Motivation Provide instances of genuine valued ou (VODs) for MCDA method debugging Discover VOD's specific structural char	 [®] [®]	Boot Boot Boot Motivation Provide instances of genuine valued outranking digraphs (VODs) for MCDA method debugging Discover VOD's specific structural characteristics Discover VOD's specific structural characteristics Comparison with other kinds of random valued digraphs	Boot Boot Boot Motivation Provide instances of genuine valued outranking digraphs (VODs) for MCDA method debugging Boot Discover VOD's specific structural characteristics Boot Comparison with other kinds of random valued digraphs Boot	Bot Bot Bot Motivation Motivation Provide instances of genuine valued outranking digraphs (VODs) for MCDA method debugging • Provide instances (VODs) for MCDA • Discover VOD's specific structural characteristics • Discover VOD's specific structural characteristics • Comparison with other kinds of random valued digraphs • Comparison with other kinds of random valued digraphs	Boot Boot Boot Boot Motivation Motivation Provide instances of genuine valued outranking digraphs (VODs) for MCDA method debugging • Provide instances of genuine valued outranking digraphs • Discover VOD's specific structural characteristics • Discover VOD's specific structural characteristics • Comparison with other kinds of random valued digraphs • Comparison with other kinds of random	Bot B		

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux 0 0 0	Random Outranking Digraphs 0 00 00	Conclusion	Motivation	Valued Outranking Digraphs 0 00 0	Random Performance Tableaux	Random Outranking Digraphs 0 00 00	Conclusion
		Motivation					Motivation		

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Motivation	Valued Outranking Digraphs oo o	Random Performance Tableaux	Random Outranking Digraphs 0 00 00	Conclusion	Motivation	Valued Outra	nking Digraphs	Random Pe	rformance Tab		Random (0 00 00	Dutranking	g Digraphs	Conclusion
		Motivation					Value	d Outi	ranking	; Dig	raph	s		
						Fact: N	lot every	valued di	graph is	a vali	d inst	ance c	of a VO	D!
	 Provide instances of genuine valued outranking digraphs (VODs) for MCDA method debugging 													
	(1000) 101 11(00)	1 1100 000066116												
	Discover VOD's s	pecific structural cha	racteristics											
	 Comparison with 	other kinds of randor	n valued digraphs											
	Mathematical cha	racterisation of VOD	s											

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux 0 00 00 0	Random Outranking Digraphs 0 00 00	Conclusion	Motivation	Valued Outranking Digraphs 0 00 0	Random Performance Tableaux	Random Outranking Digraphs	Conclusion

Valued Outranking Digraphs

Fact: Not every valued digraph is a valid instance of a VOD !

Example (bipolar valued digraphs)

Α	valid	VOD	instan	ice			
ŝ	a01	a02	a03	<i>a</i> 04			
a01	-	0.2 - 0.6 0.2	0.4	0.4			
a02	0.0	-	0.2	0.2			
a03	0.4	0.6	-	0.0			
<i>a</i> 04	-0.2	0.2	0.2	-			

 $-1.0 < (xSy) < 0.0 \quad \Rightarrow \quad (ySx) \ge 0.0 , \quad \forall x, y$

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Α	valid	VOD	instan	ce	An	invalic	VOE) insta	ance
ŝ	a01	<i>a</i> 02	<i>a</i> 03	<i>a</i> 04	ŝ	<i>a</i> 01	<i>a</i> 02	<i>a</i> 03	<i>a</i> 04
a01	- 0.0 0.4 -0.2	0.2	0.4	0.4	a01	- 0.0 0.4 -0.2	0.2	0.4	-0.3
a02	0.0	-	0.2	0.2	a02	0.0	-	0.2	0.2
a03	0.4	0.6	-	0.0	a03	0.4	0.6	-	0.0
a04	-0.2	0.2	0.2	-	<i>a</i> 04	-0.2	0.2	0.2	-

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion
	Value	d Outranking Di	graphs				valid VODs		

0 0 1

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Α	valid	VOD	instan	се	 An	invalic	VO) insta	ance
ŝ	a01	a02	a03	<i>a</i> 04	ŝ	a01	<i>a</i> 02	<i>a</i> 03	<i>a</i> 04
a01	- 0.0 0.4 -0.2	0.2	0.4	0.4	a01	- 0.0 0.4 -0.2	0.2	0.4	-0.3
a02	0.0	-	0.2	0.2	a02	0.0	-	0.2	0.2
a03	0.4	0.6	-	0.0	a03	0.4	0.6	-	0.0
<i>a</i> 04	-0.2	0.2	0.2	-	<i>a</i> 04	-0.2	0.2	0.2	-

 $-1.0 < (x\widetilde{S}y) < 0.0 \Rightarrow (y\widetilde{S}x) \ge 0.0, \quad \forall x, y$

Comments

A performance tableau shows the performances of a finite set X of decision actions on a finite set F of criteria-functions associated with significance weights and discrimination thresholds.

A valued digraph is a valid VOD iff there exists a performance tableau which generates the apparent digraph valuation.



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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	
	0000	000	0 00 00		00 0	000	0 00 00	
		Outline			The	outranking situ	ation	

Outline

Valued Outranking Digraphs

- 1.1 The outranking situation
- 1.2 The outranking index
- 1.3 The bipolar valued outranking digraph

Random Performance Tableaux

- 2.1 Reference model
- 2.2 random performances
- 2.3 random thresholds

Random Outranking Digraphs

- 3.1 Definition
- 3.2 Link densities
- 3.3 Connectivity

• Let X be a finite set of p alternatives.

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion
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	0	0	00			0	0	00	

The outranking situation

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- Let X be a finite set of p alternatives.
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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion
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- Let X be a finite set of p alternatives.
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- Let x_i be the value taken by x on criterion g_i

Definition (The outranking situation)

credibility of the validation or the managing and the

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Definition (The outranking situation)

- x outranks y (x S y) if there is a significant majority of criteria which support an at least as good statement and there is no criterion which raises a veto against it.
- The bipolar valued relation S̃ ∈ [-..., m] expresses the credibility of the volutation or the non-volidation of the outrambing relation S.

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion

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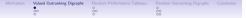
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General VOD definition - continued

Definition (The bipolar valued outranking situation)

$$\begin{split} \widetilde{S}(x,y) &= \min \left\{ \left(\sum_{i \in F} w_i \cdot C_i(x,y) \right), \min_{i \in F} \left(-V_i(x,y) \right) \cdot m \right\} \\ C_i(x,y) &= \begin{cases} 1 & \text{if } x_i + q_i > y_i; \\ -1 & \text{if } x_i + p_i \leqslant y_i; \\ 0 & \text{otherwise} \end{cases} \\ V_i(x,y) &= \begin{cases} 1 & \text{if } x_i + v_i \leqslant y_i; \\ -1 & \text{if } x_i + w_i > y_i; \\ 0 & \text{otherwise} \end{cases} \end{split}$$

where q_i, p_i represent the weak preference, resp. the preference,

and wv_i , v_i , the weak veto, resp. the veto, threshold on criterion g_i

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The VOD relation \widetilde{S} – continued

 \widetilde{S} is defined on a bipolar-valued credibility scale $\mathcal{L} = [-m, m]$ supporting the following semantics denotation:

- S(x, y) = +m means that assertion x S y is clearly validated.
- S̃(x, y) = -m means that assertion x S y is clearly non-validated.
- \$\overline{S}(x, y) > 0\$ means that assertion x S y is more validated than non-validated.
- S(x, y) < 0 means that assertion x S y is more non-validated than validated.
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VOD $\widetilde{G}(X,\widetilde{S})$

Definition (The bipolar valued outranking digraph)

- We denote G̃(X,S̃) the bipolar-valued outranking digraph modelled via S̃ on X × X.
- The associated crisp outranking relation S may be recovered from S
 as the set of pairs (x, y) such that S
 5 > 0.
- $\underline{G}(X, \underline{S})$ is called the crisp outranking digraph associated with $\overline{G}(X, \overline{S})$.

$\widetilde{G}(X,\widetilde{S})$ instance

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A G(X, S) instance

VOD $\tilde{G}(X, \tilde{S})$

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$G(X, \overline{S})$ instance

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A $\widetilde{G}(X, \widetilde{S})$ instance

ŝ	a01	<i>a</i> 02	a03	<i>a</i> 04
a01	-	0.2	0.4	0.4
a02	0.0	-	0.2	0.2
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ŝ	a01	a02	<i>a</i> 03	a04	
a01 a02	-	0.2	0.4	0.4	
	0.0	-	0.2	0.2	
a03	0.4	0.6	-	0.0	
<i>a</i> 04	-0.2	0.2	0.2	-	



Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs 0 00 00	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs 0 00 00	Conclusion	
						Ramdo	m Performance	Tableau		
	The outranking site				Definition (A reference model)					
	! The outranking index The bipolar valued outranking digraph					 20 decision actions; low variant: 13; high variant: 50. 13 criteria; low variant: 7; high variant: 20. 				
	Random Performan Reference model									
2.2 2.3	random performand random thresholds								ince	

Random Outranking Digraphs

- 3.1 Definition
- 3.2 Link densities
- 3.3 Connectivity

Ordinal performances are represented on integer scales {1, 2, ..., 10}.

 Cardinal performances are represented on a decimal scale: [0.0; 100.0] with a precision of 2 digits.

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion

Ramdom Performance Tableau

Definition (A reference model)

- 20 decision actions; low variant: 13; high variant: 50.
- 13 criteria; low variant: 7; high variant: 20.
- A criteria is with equal probability either to be minimized (cost criteria) or to be maximized (benefit criteria).
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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	
	0	•	0		0	•	0	
	0	0	00		0	0	00	

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs 0 00 00	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs 0 00 00	Conclusion

Ramdom Performance Tableau

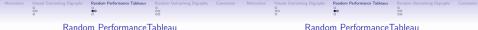
Definition (A reference model)

- 20 decision actions; low variant: 13; high variant: 50.
- 13 criteria; low variant: 7; high variant: 20.
- A criteria is with equal probability either to be minimized (cost criteria) or to be maximized (benefit criteria).
- All criteria either support an ordinal or a cardinal performance scale; the cost criteria being mostly cardinal (2/3) and the benefit ones mostly ordinal (2/3).
- Ordinal performances are represented on integer scales: {1, 2, ..., 10}.
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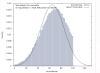
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Random PerformanceTableau

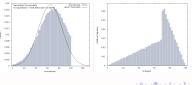
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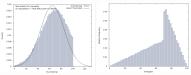




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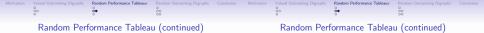
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Random Performance Tableau (continued)

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- advantageous when the performances are generated wit T(xm = 70, r = 0.5) (reference) or $N(\mu = 70, \sigma = 25)$
- and neutral when the performances are generated with
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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	
	0	0	0		0	0	0	
	00	00				00	00	
	0	•	00		0	•	00	

Random Discrimination Thresholds

On each cardinal criterion, the default discrimination thresholds are chosen such that the:

- indifference threshold equals the percentile 5 of all generated performance differences;
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- weak veto threshold equals the percentile 90 of all generated performance differences;
- veto threshold equals the percentile 95 of all generated performance differences.
- The ordinal criteria admit solely a preference threshold of one unit.

Example

Random performance tableau instance

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Example

Random performance tableau instance

			- 이미가 이름이 이렇게 이렇게	\$ DOG				- 이미가 이상 지수는 지수는 것	5 996
Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs		Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion
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(D) (B) (S) (S) (S) (S) (O)

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	
	000	00	000		000	00	000	
	0	•	00		0	•	00	

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Random performance tableau instance

			101100-001100-001100-00110-00100-00100-0010-0010-0010-0010-0010-0010-0010-0010-0010-0010-0010-0010-0010-000-000	\$ 940				(1) (1) (2) (2) (2)	\$ 990
Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion
	00	00	000			00	00	0	
	0	•	00			0	0	00	

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Random performance tableau instance

Valued Outranking Digraphs

- 1.1 The outranking situation
- 1.2 The outranking index
- 1.3 The bipolar valued outranking digraph

Random Performance Tableaux

- 2.1 Reference model
- 2.2 random performance
- 2.3 random thresholds

Random Outranking Digraphs

- 3.1 Definition
- 3.2 Link densities
- 3.3 Connectivity

Random VODs

Comments

Random VODs

Random Outranking Digraphs

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Out
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	0	0	00			0

First results: arc and link densities

Random Outranking Digraphs

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Random VODs

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- The sample size 3000 associates a 99% confidence to almost all average characteristics of the sample.
- When the standard deviation is less than 10%, the confidence interval around mean percentage results is thus less than 0.5%.

• For the reference ROD we observe in average:

- an arc density of 47.9%(4.4)
- a double link density of 14.22%(4)
- a single link density of 67.60%(6)
- a link absence density of 18.19%(6)
- A similar sample of standard random digraphs (SRDs) with an arc probability of 48% would show a binomial probability of single links (46%), double links (25%) and absence of links (27%).
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	0	0	0			0	0	0	
	00	00	00				00		
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	0	0	0		0	0	0	
	00	00	00		00	00	00	
	0	0	00		0	0	00	

First results: more or less vetoes

 The number of effectively raised vetoes is directly related to the average number of cardinal criteria we observe in the ROD.

link type			13 crit average (7 criteria average (stdev.)	
arc	40.0%	(4.2)	47.9%	(4.4)	52.2%	(3.7)
double	9.8%	(3.0)	14.22%	(4.1)	13.9%	(4.0)
single	60.5%	(7.0)	67.60%	(6.1)	76.6%	(5.2)
absence	29.7%	(7.3)	18.19%	(6.4)	9.4%	(4.9)

 Without any vetoes, we are faced with bipolar-valued weak tournaments where there are always either a single or a double link between all pairs (x, y) of actions:

$(x\widetilde{S}y) + (y\widetilde{S}x) \ge 0.0$, $\forall x, y$

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion
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First results: graph connectivity

• RODs, as well as SRDs always show one single component.

- SRDs nearly always (99%) show a single strong component.
- RODs, however, may show up to 8 strong components

Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion
	0	0	0			0	0	0	
	0	0	0			0	0	0	

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ROD frequency distribution of multiple strong components

nbr.	frequency	rel.	cum.	leaves
1	1210	40.33%	40.33%	*****
2	955	31.83%	72.17%	******
3	517	17.23%	89.40%	*****
4	203	6.77%	96.17%	**
5	72	2.40%	98.57%	
6	26	0.87%	99.43%	
7	13	0.43%	99.87%	
8	4	0.13%	100.00%	

First results: multiple vetoes and strong components

Even with 20 criteria, RODs show always one single

component. But, up to 11 strong components (in a digraph of order 20) may now appear:

		*

Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion
	0	0	0		0	0	0	
	00	00				00	00	
	0	0	00		0	0	00	

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nbr.	frequency	rel.	cum.	leaves
1	635	21.17%	21.17%	******
2	809	26.97%	48.13%	*******
3	648	21.60%	69.73%	******
4	423	14.10%	83.8%	****
5	253	8.43%	92.27%	***
6	117	3.90%	96.17%	*
7	60	2.00%	98.17%	
8	28	0.93%	99.10%	
9	19	0.63%	99.73%	
10	5	0.17%	99.90%	
11	3	0.10%	100.00%	

Concluding Remarks

In this communication we have presented:

- Generators for random performance tableaux
- A reference model for random outranking digraphs
- Some empirical statistical results

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Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion	Motivation	Valued Outranking Digraphs	Random Performance Tableaux	Random Outranking Digraphs	Conclusion
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References I

Bisdorff R (2002)

Logical Foundation of Multicriteria Preference Aggregation.

In: Bouyssou D et al (eds) *Essay in Aiding Decisions with Multiple Criteria.* Kluwer Academic Publishers 379–403

R. Bisdorff (2008)

The Python digraph implementation for RuBis: User Manual. University of Luxembourg, http://ernst-schroeder.uni.lu/Digraph.

B. Bollobás (2001) Random Graphs.

Cambridge University Press.

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