Random Outranking Digraphs

Raymond Bisdorff (*and Bernard Roy*)

University of Luxembourg, FSTC/CSC

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Motivation

- Provide instances of genuine valued outranking digraphs (VODs) for MCDA method debugging
- Discover VOD's specific structural characteristics
- Comparison with other kinds of random valued digraphs
- Mathematical characterisation of VODs

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Valued Outranking Digraphs

Fact: Not every valued digraph is a valid instance of a VOD !

Example (bipolar valued digraphs)

A valid VOD instance

$\widetilde{S}$	$a01$	$a02$	$a03$	$a04$
$a01$	-	0.2	0.4	0.4
$a02$	0.0	-	0.2	0.2
$a03$	0.4	0.6	-	0.0
$a04$	-0.2	0.2	0.2	-

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$$-1.0 < (x\widetilde{S}y) < 0.0 \quad \Rightarrow \quad (y\widetilde{S}x) \geqslant 0.0\,, \quad \forall x, y$$

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valid VODs

Comments

A performance tableau shows the performances of a finite set X of decision actions on a finite set F of criteria-functions associated with significance weights and discrimination thresholds.

A valued digraph is a valid VOD iff there exists a performance tableau which generates the apparent digraph valuation.

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Outline

The outranking situation

- Let X be a finite set of p alternatives.
- Let F be a finite set of $n > 1$ criteria.
- Let m be the total significance of the criteria.
- Let x and y be two alternatives from X.
- Let x_i be the value taken by x on criterion g_i

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Definition (The outranking situation)

- x outranks y ($x \, \mathrm{S} \, y$) if there is a significant majority of criteria which support an at least as good statement and there is no criterion which raises a veto against it.
- The bipolar valued relation $\widetilde{\mathrm{S}} \in [-m, m]$ expresses the credibility of the validation or the non-validation of the outranking relation S.

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General VOD definition – continued

Definition (The bipolar valued outranking situation)

$$\widetilde{\mathrm{S}}(x,y) = \min \Big\{ \Big(\sum_{i \in F} w_i \cdot C_i(x,y) \Big), \min_{i \in F} \big(- V_i(x,y) \big) \cdot m \Big\}$$

$$C_i(x,y) = \begin{cases} 1 & \text{if} \quad x_i + q_i > y_i; \\ -1 & \text{if} \quad x_i + p_i \leqslant y_i; \\ 0 & \text{otherwise} \end{cases}$$

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where q_i, p_i represent the weak preference, resp. the preference, and wv_i, v_i, the weak veto, resp. the veto, threshold on criterion g_i.

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The VOD relation $\widetilde{\mathrm{S}}$ – continued

$\widetilde{\mathrm{S}}$ is defined on a bipolar-valued credibility scale $\mathcal{L} = [-m, m]$ supporting the following semantics denotation:

- $\widetilde{\mathrm{S}}(x,y) = +m$ means that assertion $x\,\mathrm{S}\,y$ is clearly validated.
- $\widetilde{\mathrm{S}}(x,y) = -m$ means that assertion $x\,\mathrm{S}\,y$ is clearly non-validated.
- $\widetilde{\mathrm{S}}(x,y) > 0$ means that assertion $x\,\mathrm{S}\,y$ is more validated than non-validated.
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VOD $\widetilde{G}(X, \widetilde{S})$

Definition (The bipolar valued outranking digraph)

- We denote $\widetilde{G}(X, \widetilde{S})$ the bipolar-valued outranking digraph modelled via $\widetilde{S}$ on $X \times X$.
- The associated crisp outranking relation S may be recovered from $\widetilde{S}$ as the set of pairs (x,y) such that $\widetilde{S} > 0$.
- $G(X, S)$ is called the crisp outranking digraph associated with $\widetilde{G}(X, \widetilde{S})$.

A $\widetilde{G}(X, \widetilde{S})$ instance

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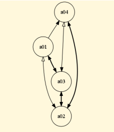

Rubis Python Server (graphviz), R. Bisdorff, 2008

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Ramdom Performance Tableau

Definition (A reference model)

- 20 decision actions; low variant: 13; high variant: 50.
- 13 criteria; low variant: 7; high variant: 20.
- A criteria is with equal probability either to be minimized (cost criteria) or to be maximized (benefit criteria).
- All criteria either support an ordinal or a cardinal performance scale; the cost criteria being mostly cardinal (2/3) and the benefit ones mostly ordinal (2/3).
- Ordinal performances are represented on integer scales: $\{1, 2, \ldots, 10\}$.
- Cardinal performances are represented on a decimal scale: $[0.0; 100.0]$ with a precision of 2 digits.

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- 13 criteria; low variant: 7; high variant: 20.
- A criteria is with equal probability either to be minimized (cost criteria) or to be maximized (benefit criteria).
- All criteria either support an ordinal or a cardinal performance scale; the cost criteria being mostly cardinal (2/3) and the benefit ones mostly ordinal (2/3).
- Ordinal performances are represented on integer scales: $\{1, 2, ..., 10\}$.
- Cardinal performances are represented on a decimal scale: $[0.0; 100.0]$ with a precision of 2 digits.

Motivation Valued Outranking Digraphs Random Performance Tableaux Random Outranking Digraphs Conclusion

Ramdom Performance Tableau

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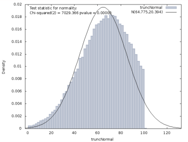
Random PerformanceTableau

Three random performance generators may be considered:

- a truncated normal generator ($\mathcal{N}(\mu, \sigma)$);
- a triangular generator ($\mathcal{T}(xm, r)$) with mode xm and probability repartition r;
- and a uniform generator ($\mathcal{U}(0.0, 100)$).



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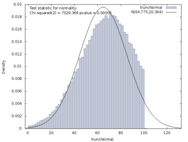

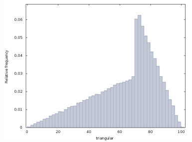

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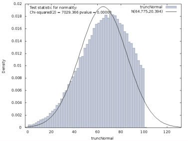

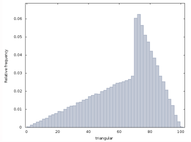

Random Performance Tableau (continued)

- In the reference model the decision actions are divided randomly into three categories: *cheap*, *neutral*, *advantageous*.
- An action is called:
  - cheap when the performances are generated with $\mathcal{T}(xm = 30, r = 0.5)$ (reference) or $\mathcal{N}(\mu = 30, \sigma = 25)$,
  - advantageous when the performances are generated with $\mathcal{T}(xm = 70, r = 0.5)$ (reference) or $\mathcal{N}(\mu = 70, \sigma = 25)$,
  - and [illegible] when the performances are generated with $\mathcal{T}(xm = 50, r = 0.5)$ (reference) or $\mathcal{N}(\mu = 50, \sigma = 25)$.

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Random Discrimination Thresholds

On each cardinal criterion, the default discrimination thresholds are chosen such that the:

- indifference threshold equals the percentile 5 of all generated performance differences;
- preference threshold equals the percentile 10 of all generated performance differences;
- weak veto threshold equals the percentile 90 of all generated performance differences;
- veto threshold equals the percentile 95 of all generated performance differences.

- The ordinal criteria admit solely a preference threshold of one unit.

Example

Random performance tableau instance

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Random VODs

Comments

- *From the previous reference random performance tableaux, we are going to generate a randomly valued outranking digraph $\widetilde{G}(X, \widetilde{S})$.*
- *We call ROD a sample of 3000 such valued outranking digraphs obtained from independently sampled random performance tableaux.*
- *The sample size 3000 associates a 99% confidence to almost all average characteristics of the sample.*
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First results: arc and link densities

- For the reference ROD we observe in average:
  - an arc density of 47.9%(4.4);
  - a double link density of 14.22%(4);
  - a single link density of 67.60%(6);
  - a link absence density of 18.19%(6).
- A similar sample of standard random digraphs (SRDs) with an arc probability of 48% would show a binomial probability of single links (46%), double links (25%) and absence of links (27%).
- As it is the case of SRDs, these results are in fact independent of the order of the RODs.

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First results: more or less vetoes

- The number of effectively raised vetoes is directly related to the average number of cardinal criteria we observe in the ROD.

link type	20 criteria average	(stdev.)	13 criteria average	(stdev.)	7 criteria average	(stdev.)
arc	40.0%	(4.2)	47.9%	(4.4)	52.2%	(3.7)
double	9.8%	(3.0)	14.22%	(4.1)	13.9%	(4.0)
single	60.5%	(7.0)	67.60%	(6.1)	76.6%	(5.2)
absence	29.7%	(7.3)	18.19%	(6.4)	9.4%	(4.9)

- Without any vetoes, we are faced with bipolar-valued weak tournaments where there are always either a single or a double link between all pairs (x, y) of actions:

$$\boxed{(x\widetilde{S}y) + (y\widetilde{S}x) \geqslant 0.0\,, \quad \forall x, y}$$

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First results: graph connectivity

- RODs, as well as SRDs always show one single component.
- SRDs nearly always (99%) show a single strong component.
- RODs, however, may show up to 8 strong components.

ROD frequency distribution of multiple strong components

nbr.	frequency	rel.	cum.	leaves
1	1210	40.33%	40.33%	**************
2	955	31.83%	72.17%	***********
3	517	17.23%	89.40%	******
4	203	6.77%	96.17%	**
5	72	2.40%	98.57%	
6	26	0.87%	99.43%	
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First results: multiple vetoes and strong components

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nbr.	frequency	rel.	cum.	leaves
1	635	21.17%	21.17%	*******
2	809	26.97%	48.13%	*********
3	648	21.60%	69.73%	*******
4	423	14.10%	83.8%	*****
5	253	8.43%	92.27%	***
6	117	3.90%	96.17%	*
7	60	2.00%	98.17%	
8	28	0.93%	99.10%	
9	19	0.63%	99.73%	
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Concluding Remarks

In this communication we have presented:

- Generators for random performance tableaux
- A reference model for random outranking digraphs
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