Re-reading the user manual

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1 Introduction

"... The goal of our research was to design a resolution method ... that is easy to put into practice, that requires as few and reliable hypotheses as possible, and that meets the needs [of the decision maker]..."[1] BRS[2].

While accepting with enthusiasm the invitation of the editors to comment on Bernard Roy's first unpublished report on the ELECTRE method [1], it took me, however, quite some time in order to find the right words to do so. As a Lamsade PhD student of Roy from 1976 to 1981, I was at that time so embedded in the ongoing development of the new ELECTRE methods for multiple criteria based decision aiding, that this seminal report on the outranking concept with its first application to the best choice decision problem remained for me hidden in the protohistory of the outranking methods. The early Lamsade period at the end of the seventies – ten years after this very first report – was a major productive period in the elaboration of the whole family of the nowadays classical ELECTRE decision aid methods [2; 3]. Reading at present this early text produces in my mind the same effect that happens when one re-reads, as a meanwhile experienced user, the detailed user manual of some complicated device. One generally gets a lot of confirmation of the actual understanding one has gained meanwhile of its genuine operating conditions. However, one also might get unexpected new insights which, the case given, may enhance ones future operating performance. This happened indeed to me when reading the founding text of the outranking concept and the ELECTRE best choice selecting aid method. Here I would like to shortly report both, on some of these confirming insights like Roy's fundamental methodological pragmatism and, on a new insight I have experienced and that concerns the operational difficulties inherent in the practical implementation of

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[1] " *... L'objectif des recherches était de mettre au point une méthode de résolution ... qui soit facile à utiliser, qui nécessite des hypotèses simples, aussi peu nombreuses et peu contestables que possible et qui puisse répondre aux besoins...*"

[2] Benayoun R, Roy B, Sussmann B. ELECTRE : *Une méthode pour guider le choix en présence de points de vue multiples*, Société d'Economie et de Mathématique Appliquée, Direction Scientifique. Note de Travail n°49, Juin 1966, avant-propos

the discordance principle. To illustrate this new insight, I will in a third section re-solve Roy's original illustrative decision problem with the Rubis best choice method [4], a recently published follow-up of the classical ELECTRE IS method.

2 Confirming Roy's pragmatism

> *"... The* ELECTRE *method is above all ... a selecting method. It was invented in order to support a choice decision in a* simple way *[we emphasize], i.e. with a help of a simple mechanism whose functioning is constantly controllable, and requiring as less restrictive hypotheses as possible... "* [3] BRS op.cit. p.11.

My first attraction to Roy's decision aid methodology stems from the fact that his approach is rooted in a sound pragmatism, i.e. in "*a way of dealing with problems in a sensible, practical way instead of following a set of ideas*"[4]. When proposing a specific resolution principle for the unique best choice decision problem, the apparent decision aid approach is clearly based on "*the doctrine that practical consequences are the criteria of knowledge and meaning and value*"[5]. By the time this methodological premise has become the major distinctive feature of all further outranking based decision aid methods [2; 3]. Indeed, the classical optimisation approach of the mathematical programming community at that time, still very much apparent in Roy's contemporary graph theoretical text books [5; 6] for instance, is quite different. This major shift of attention towards a real pragmatic assistance for the actors of a decision making process represents the originality of modern decision aid theory in contrast with the classical Operations Research field. A new scientific subject – multiple criteria decision aid – is born here, which will prove its viability and specificity until now.

Roy's seminal decision theoretical contribution is also very different from the decision making analysis of the classical decision sciences field. The utility or value based decision making analysis [7] is modelling rational, i.e. optimal or satisfactory, decisions via the construction of commensurable evaluations. To clearly state its opposition in the seminal text, the illustrative decision alternatives' performances on the preference points of view are purposefully evaluated on purely ordinal scales[6]. Indeed, in his didactic example, Roy will be using throughout the same preference scale composed of 5 linguistic grades : *bad*, *weak*, *average*, *good*, and *excellent* on all preferential points of view, i.e. criteria in modern decision aid terms.

Preserving above all the meaningfulness of the preferential information treatment appears to have been from the beginning on a major concern of Roy's

[3] *"... La méthode [*ELECTRE*] est avant tout ... une méthode visant à la sélection. Elle a été conçue pour permettre d'éclairer un choix, et ceci de façon simple, c'est-à-dire à l'aide d'un mécanisme simple, dont le fonctionnement est constamment contrôlable, et en faisant appel à des hypothèses aussi peu restrictives que possible ..."*

[4] Longman Dictionary of Contemporary English

[5] WordNet

[6] *"[La méthode* ELECTRE*] ... ne nécessite pas de faire des additions de jugements, appréciations de notes attribuées suivant des échelles [de mesure] différentes"*. The ELECTRE method ... does not require any additional judgments, appreciations of grades from different [measurement] scales. *BRS op.cit. p.11.*

decision aid approach. But, abandoning, thus, any hope to be able to construct commensurable utilities or value functions presents a severe operational and computational drawback. The aggregation of essentially ordinal preferences generally observed on multiple preferential points of views becomes now a challenging and non trivial problem that will be tackled with the help of the innovative *outranking* concept.

3 The epistemic foundation of the outranking concept

> *"On what principles may we base the arcs to be added to G_0 [the unanimous dominance digraph] in order to deduce a digraph G which is both – in best possible concordance with the [given] multiple [preferential] points of view and – not too poor in comparable pairs [of decision actions] ?* [7] *"* BRS op.cit. p.16

Ideas from social choice theory gave initially the insight that pairwise voting mechanism à la Condorcet could provide an order-statistical tool for aggregating a set of preference points of views into what Barbut calls the central *Condorcet* point of view[8] [8]. Considering thus each criterion as a subset of unanimous voters and arithmetically counting the votes in favour of a given preference statement over all criteria gives rise to a second distinctive feature of the multicriteria decision aid field, namely the *outranking* situation. A modern definition would be : An alternative x is said to "*outrank*" alternative y if and only if – there exists a *significant* majority[9] of criteria who confirm that alternative x is to be considered as *at least as worth as* alternative y (*concordance* principle) ; and – no discordant criterion opens to significant doubt the validity of the previous confirmation by revealing a serious counter-performance of alternative x compared to y (*discordance* principle).

This concordance–discordance definition consists essentially in balancing affirmative against refutative preferential reasons for judging the validity of a given pairwise "at least as worth" statement [10]. Inspired originally by the contemporary social choice theory [11; 8; 12], the definition and semiotics of the significantly concordant part of the outranking definition, i.e. the so-called concordance index of the outranking situation, will in the sequel be largely accepted in the decision aid community [10] [10]. The same positive reception will, however, not happen with the original discordant part of the outranking defi-

[7] "*En vertu de quels principes peut-on caractériser les arcs qu'il convient d'adjoindre à G_0 [Le graphe de dominance unanime] pour en déduire un graphe G qui soit en aussi bon accord que possible avec les différents points de vue, sans être pour cela trop pauvre en couples comparables ?*

[8] In fact the median of the multiple preference points of view at minimal absolute Kendall τ measure from all individual points of view.

[9] "*une majorité suffisante*" BRS op.cit. p.21.

[10] "*Cet indicateur s'interprète facilement, en considérant les points de vue comme des votants plus ou moins représentatifs : c'est le pourcentage des voix en faveurs de l'hypothèse* [This index is easily understood when considering the points of views like more or less representative voters : it is the pourcentage of votes in favour of the hypothesis]." BRS op.cit. p.17.

nition. Based in the seminal text on the commensurable ordinal ranking of the preference differences observed on all the points of view, the first discordance index implementation appears strange to a modern reader, somehow even in contradiction with the otherwise claimed simplicity of the ELECTRE method.

The proposed performance tableau concept is indeed convincingly objective and meaningful, and so appears also the proposed outranking concordance index. The seminal text fails however to provide a similar objective and pragmatical foundation of the discordance principle. And this remains indeed one of the controversial features of the outranking based decision aid approaches. The Promethee outranking approach for instance will simply ignore the discordant part in its single net flow matrix based approach [13]. We have also shown more explicitly that the original concordance index is formally identical with a majority margins index, where the affirmative criteria are taking positively (the concordant part) and the refutative ones (the discordant part) are taking negatively [9; 10]. Hence, we get a bipolar valued concordance index – valued from -1 to 1 –, with the 0 value acting as indeterminate value, that is order-isomorphic to the initially proposed concordance index valued in a unipolar rational domain from 0 to 1.

Is it therefore really necessary to boost the discordance part of the outranking concept by introducing what we now call the *veto* principle ? In defense of the seminal outranking definition, we may, however, quote Pirlot and Vincke : "*... If [decision alternative] y is so much better than [decision alternative] x on some criteria, in spite of a [significant] majority [of criteria] supporting x, it seems wise not to conclude ...*"[14].

The seminal text provides a fully worked out didactic example which is supposed to demonstrate among other the very usefulness of the discordance part of the new outranking definition. We will therefore, in the next section, revisit this example from a modern perspective.

4 Re-solving Roy's first didactic best choice problem

Those who have worked with Bernard Roy easily recognize the acribic choice of his didactic examples. The 1966 methodological text is similarly illustrated with the help of a small – 6 alternatives evaluated on 5 criteria – but completely worked out best choice decision problem. Even after more than 40 years of intense research and methodological progress on such kind of decision problems [2; 3; 4], the seminal example remains worth being analysed.

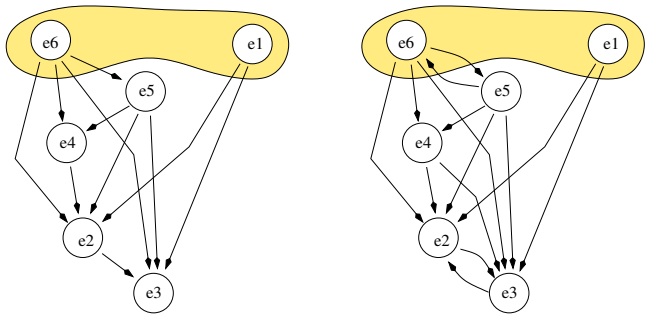
The illustrative decision problem concerns the selection of the best from a set of six objects : $A = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, appreciated on five preferential points of view $\{p_1, p_2, p_3, p_4, p_5\}$ which are supposed to be weighted as follows : $w_1 = w_3 = 3$, $w_2 = 2$ and $w_4 = w_5 = 1$. The preferential appreciations are represented on an ordinal preference scale with 5 linguistic grades : *bad*, *weak*, *average*, *good*, and *excellent*. The proposed performances of all alternatives on

each criterion are shown in Table 1.

TAB. 1 – The performance table

Criteria	e_1	e_2	e_3	e_4	e_5	e_6
p_1	average	bad	bad	excellent	excellent	excellent
p_2	excellent	weak	average	weak	average	average
p_3	weak	weak	bad	average	good	excellent
p_4	average	excellent	excellent	average	average	good
p_5	excellent	average	weak	average	good	good

It is worthwhile noticing that the performances of alternatives e_6, e_5 and e_4 give an unanimous global outranking order $e_6 \geqslant e_5 \geqslant e_4$. From the concordance index when ignoring any potential vetoes, we get a Condorcet outranking digraph showing a complete weak order : $\{e_6, e_5\} \geqslant e_4 \geqslant e_1 \geqslant \{e_2, e_3\}$. When requiring a two third majority concordance for validating an outranking situation, we get the complete order $e_6 \geqslant e_5 \geqslant e_4 \geqslant e_1 \geqslant e_2 \geqslant e_3$. Finally, with a three fourth majority we get the outranking digraph shown in Figure 1 (a).



(a) Best choice recommendation from 75% concordance without vetoes

(b) Best choice recommendation from variant (2) veto thresholds

FIG. 1 – Outranking digraphs with best choice recommendations

Following the operational requirements of the best unique choice decision problematique, the outranking kernel reveals here the pair (e_6, e_1) as potential ELECTRE best choice recommendation. The decision aid solution makes sense as it retains both e_6 – certainly dominating e_5, e_4, e_2, and e_3 –, and e_1, which compares badly with e_6 at this high significance level of the concordant outranking situation. The best choice recommendation appearing here without having to invoke any obscure value based discordance principle, is identical to the seminal recommendation. May we therefore safely ignore the discordant part of the outranking definition ?

Implementing the discordance principle via the preferential discrimination tuning of the criteria functions was originally, and remains also nowadays, a not trivial preference modelling problem. Following here as close as possible the original text, we propose in Table 2 modern type criterion-functions with

potentially equivalent ELECTRE-type veto thresholds. We distinguish two vari-

TAB. 2 – The coded performance tableau

Criteria	w	Performances						scale	Veto thresholds	
		e_1	e_2	e_3	e_4	e_5	e_6		variant 1	variant 2
p_1	3	10	0	0	20	20	20	0–20	11	7
p_2	2	20	5	10	5	10	10	0–20	11	7
p_3	3	5	5	0	10	15	20	0–20	11	7
p_4	1	10	16	16	10	10	13	4–16	11	7
p_5	1	16	10	7	10	13	13	4–16	11	7

ants : – a weak (1) and a strong (2) set – of veto thresholds. In variant (1), a counter-performance of *weak* instead of *excellent* or *bad* instead of *good* on the more significant points of view p_1,p_2 and p_3, opens to doubt an otherwise significantly concordant outranking. On the lesser significant criteria, the same happens only with a counter-performance of *bad* instead of *excellent*. In variant (2), a counter-performance of more than two ordinal degrees on the more significant, and of more than three degrees on the lesser significant criteria, triggers the discordance principle.

With variant (1), only the single outranking situation observed between alternatives e_4 and e_1 is opened to doubt in the otherwise complete outranking preorder $\{e_6, e_5\} \succcurlyeq e_4 \succcurlyeq e_1 \succcurlyeq \{e_2, e_3\}$. With variant (2), however, the strong veto thresholds reduce the initial Condorcet outranking relation to the partial preorder observed in Figure 1 (b). The best choice recommendation gathers again alternatives e_6 and e_1. The first one is recommended as it outranks all the remaining alternatives with the exception of e_1 which in turn appears incomparable with e_6, e_5 and e_4.

It is important to realize at this point the quite similar outranking picture we get with the previous strong majority concordance threshold of 75% and with the strong veto thresholds (compare Figures 1(a) and 1(b)). Rising the majority threshold for validating a concordant outranking *or* stressing the veto thresholds give, hence, in the seminal illustrative example at least, the same solution, namely the best choice recommendation $\{e_6, e_1\}$ and this, with identical preferential arguments.

Let us conclude this section by mentioning that the resolving of the illustrative example has been realized with the Decision-Deck based Rubis web service[11].

5 A new insight for concluding

The results obtained are far from being anecdotal. Rising, on the one hand, the sufficient majority threshold for validating a concordant outranking, or,

[11] Interested readers might consult the Decision-Deck Rubis web services (see http ://ernst-schroeder.uni.lu/d3/ user : demo pswd : D3_Demo) in order to experiment the effective resolution of different variants of Roy's seminal best choice decision problem.

stressing, on the other hand, the veto thresholds, are in fact triggering basically the same discordance principle. In the limit, these preference modelling approaches produce in their weak expression an outranking digraph not much different from the basic Condorcet digraph. In their strongest expressions, similarly, they both tend to the same unanimous dominance digraph. Hence, these two preference modelling approaches are somehow redundant in their discordance capturing effect. Either of both is therefore potentially sufficient for implementing alone the required counterbalancing discordant part of the outranking definition.

On may guess that the choice of weighting positively the concordant criteria whereas at the same time ignoring the otherwise discordant criteria gave the authors of the seminal text the false conviction that the resulting Condorcet outranking relation should always be counter-weighted with a kind of veto based discordance principle. This veto principle, being strange, however, with the order-statistical foundation of the concordance principle, has not found an equally positive reception than the concordance principle in the multiple criteria decision analysis community [9]. Indeed, appreciating in a practical decision aid process such veto thresholds represents one of the cognitively difficult tasks which are required from the decision-maker when using an outranking based decision aid method.

In the introduction we mentioned that new insights from re-reading an excellent user manual, like the seminal text discussed here, may potentially enhance our operational performance. Following our previous interpretation, we recognize indeed in the end, that we have in fact the opportunity to adopt either of both ways of modelling the discordance part of the outranking concept. Sufficiently rising the required majority concordance validation gives indeed a potentially equivalent, but much easier, discordance stressing approach, than having to fix adequate veto thresholds.

Références

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